Second degré

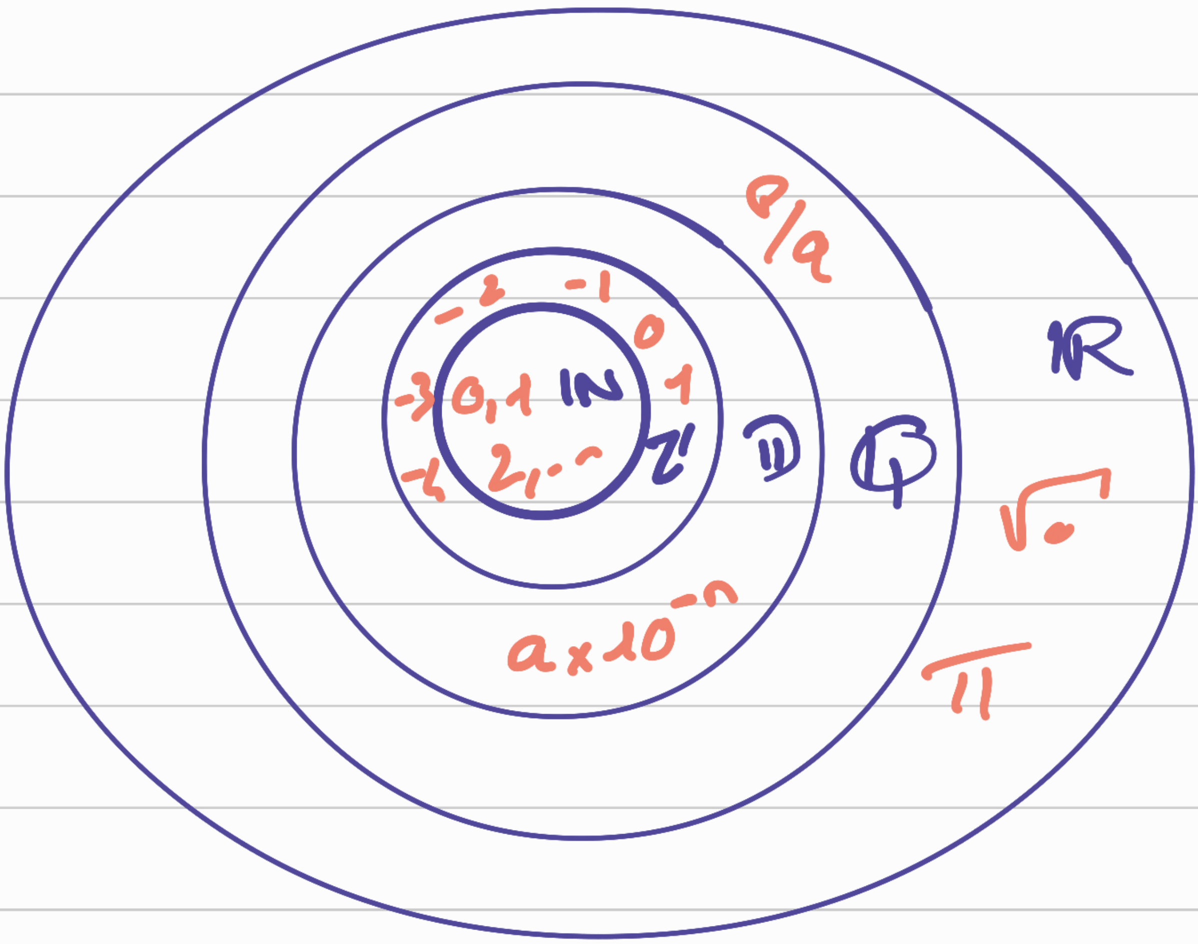
1 - Forme canonique

$$f(x) = ax^2 + bx + c$$

avec $a, b, c \in \mathbb{R}$

↳ appartient

remarque : $\mathbb{R} = \{$ tous les nombres $\}$



$$\boxed{f(x) = a(x-\alpha)^2 + \beta \quad \text{avec} \begin{cases} \alpha = \dfrac{-b}{2a} \\ \beta = f(\alpha) \end{cases}}$$

↳ forme canonique

<u>remarque</u> : • $f(x) = 2x + 3$

$f(2) = 2 \times 2 + 3 = 7$

↳ image de 2 par f

• $g(x) = 3x^2 - 2x + 5$

$g(2) = 3 \times 2^2 - 2 \times 2 + 5 = 13$

__exemple__ : $g(x) = \underbrace{3}_{a}x^2 \underbrace{-2}_{b}x + \underbrace{5}_{c}$

Donner la forme canonique.

- $\alpha = -\dfrac{b}{2a} = -\dfrac{(-2)}{2\times 3} = \dfrac{2}{6} = \dfrac{1}{3}$

- $\beta = g\left(\dfrac{1}{3}\right) = 3\times\left(\dfrac{1}{3}\right)^2 - 2\times\dfrac{1}{3} + 5$

$$= \cancel{3}\times\frac{1}{\underbrace{9}_{\cancel{3}\times 3 \to 9}} - \frac{2}{3} + 5$$

$$= \boxed{\frac{1}{3} - \frac{2}{3}} + 5$$

$$= -\frac{1}{3} + 5$$

$$= \frac{-1 + 3\times 5}{3}$$

$$= \frac{14}{3}$$

donc la forme canonique :

$$g(x) = 3\left(x - \frac{1}{3}\right)^2 + \frac{14}{3}.$$

<u>**exemple**</u> : $f(x) = \underset{a}{2}x^2 - \underset{b}{3}x + \underset{c}{4}$

- $\alpha = -\dfrac{b}{2a} = \dfrac{-(-3)}{2\times 2} = \dfrac{3}{4}$

- $\beta = f\left(\dfrac{3}{4}\right)$

$$= 2\times\left(\frac{3}{4}\right)^2 - 3\times\frac{3}{4} + 5$$

$$= \cancel{2}\times\frac{9}{\underset{\cancel{2}\times 8}{16}} - \frac{9}{4} + 5$$

$$= \frac{9}{8} - \frac{9}{4} + 5$$

$$= \frac{9}{8} - \frac{9_{\times 2}}{4_{\times 2}} + 5$$

$$= \frac{9}{8} - \frac{18}{8} + 5$$

$$= -\frac{9}{8} + 5$$

$$= \frac{-9 + 5 \times 8}{8}$$

$$= \frac{-9 + 40}{8}$$

$$= \frac{31}{8}$$

donc :

$$f(x) = 2\left(x - \frac{3}{4}\right)^2 + \frac{31}{8}$$

2 - Forme factorisée

$f(x) = ax^2 + bx + c$

avec $a, b, c \in \mathbb{R}$.

- $\Delta = b^2 - 4ac$

$\Delta < 0$ → pas de factorisation

$\Delta = 0$ → $x_0 = -\dfrac{b}{2a}$ → $f(x) = a(x - x_0)^2$

$\Delta > 0$ → $\begin{cases} x_1 = \dfrac{-b - \sqrt{\Delta}}{2a} \\ x_2 = \dfrac{-b + \sqrt{\Delta}}{2a} \end{cases}$ → $f(x) = a(x - x_1)(x - x_2)$

exemple : $f(x) = \underset{a}{2}x^2 \underset{b}{-3}x + \underset{c}{4}$

$$\Delta = b^2 - 4ac = (-3)^2 - 4 \times 2 \times 4$$
$$= 9 - 32$$
$$= -23$$
$$< 0$$

$$\bullet\ g(x) = \underbrace{-5}_{a}x^2 \underbrace{-9}_{b}x + \underbrace{2}_{c}$$

$$\Delta = (-9)^2 - 4 \times -5 \times 2$$

$$= 81 + 40$$

$$= 121 > 0$$

$$x_1 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{-(-9) - \sqrt{121}}{2 \times -5}$$

$$= \frac{9 - 11}{-10}$$

$$= \frac{-\cancel{2}}{\underset{\cancel{2} \times 5}{-10}} = \frac{1}{5}$$

$$x_2 = \frac{-b+\sqrt{\Delta}}{2a}$$

$$= \frac{-(-9)+\sqrt{121}}{2\times -5}$$

$$= \frac{9+11}{-10}$$

$$= \frac{20}{-10}$$

$$= -2$$

donc :

$$g(x) = -5\left(x-\frac{1}{5}\right)\left(x-(-2)\right)$$

$$= -5\left(x-\frac{1}{5}\right)(x+2).$$

exemple : $f(x) = 1 \times x^2 + 2x - 3$

$\Delta = b^2 - 4ac = 2^2 - 4 \times 1 \times -3$

$= 16$

$x_1 = -3$ $\qquad$ $x_2 = 1$

$f(x) = (x+3)(x-1)$.