Intégration

exercice 58 :

IPP :

$\rightarrow \underset{D}{x^n} \; \underset{P}{e^x}$

$\rightarrow \underset{P}{x^n} \; \underset{D}{\ln(x)}$

1) $I = \displaystyle\int_0^{\pi} \overset{D}{e^x} \, \overset{P}{\sin(x)} \, dx$

$= \left[e^x \times -\cos(x) \right]_0^{\pi} - \displaystyle\int_0^{\pi} e^x \times -\cos(x) \, dx$

$$= \left(e^{\pi} \times -\cos(\pi) - e^{0} \times -\cos(0)\right)$$

$$+ \int_{0}^{\pi} e^{x} \cos(x)\, dx$$

$$= e^{\pi} + 1 + \underbrace{\int_{0}^{\pi} e^{x} \cos(x)\, dx}_{J}$$

$$J = \int_{0}^{\pi} e^{x} \cos(x)\, dx$$

$$= \underbrace{\left[e^{x} \sin(x)\right]_{0}^{\pi}}_{=0} - \int_{0}^{\pi} e^{x} \sin(x)\, dx$$

$$= -\int_{0}^{\pi} e^{x} \sin(x)\, dx$$

$$= -I$$

2)

$$I = e^{\pi} + 1 - I$$

$$2I = e^{\pi} + 1$$

$$I = \frac{e^{\pi} + 1}{2}.$$

remarque :

$$I = \int_0^{\pi} e^{x} \sin(x)\, dx$$

or $\sin(x) = \mathrm{Im}(e^{ix})$

$$\int_0^{\pi} e^{x} \times e^{ix} \, dx$$

$$= \int_0^{\pi} e^{(1+i)x} \, dx$$

$$= \left[\frac{1}{1+i} e^{(1+i)x} \right]_0^{\pi}$$

$$= \frac{1}{1+i} \left(e^{(1+i)\pi} - e^{(1+i)0} \right)$$

$$= \frac{1}{1+i} \left(e^{(1+i)\pi} - 1 \right)$$

$$= \frac{1-i}{2} \left(e^{(1+i)\pi} - 1 \right)$$

$$= \frac{1-i}{2}\left(\underbrace{e^{(1+i)\pi}}_{e^{\pi}\times \underbrace{e^{i\pi}}_{-1} = -e^{\pi}} - 1\right)$$

$$= \overbrace{\frac{1}{2}\left(-e^{\pi}-1\right)}^{Re}$$

$$\underbrace{-\frac{i}{2}\left(-e^{\pi}-1\right)}_{Im}.$$

donc $$I = -\frac{1}{2}\left(-e^{\pi}-1\right)$$

$$= \frac{e^{\pi}+1}{2}.$$

exercice 59 :

1) $$I_0 = \int_0^{\pi/2} \cos^0(x)\, dx$$

$$= \int_0^{\pi/2} 1 \, dx$$

$$= \left[x \right]_0^{\pi/2}$$

$$= \frac{\pi}{2}.$$

$$I_1 = \int_0^{\pi/2} \cos(x)\,dx$$

$$= \Big[\sin(x)\Big]_0^{\pi/2}$$

$$= \sin\left(\frac{\pi}{2}\right) - \underbrace{\sin(0)}_{=0}$$

$$= 1$$