

Intégration

exercice 58 :

IPP :

$$\rightarrow x^n e^x$$

(Red arrow from x^n to D , Green arrow from e^x to P)

$$\rightarrow x^n \ln(x)$$

(Green arrow from x^n to P , Red arrow from $\ln(x)$ to D)

1) $I = \int_0^{\pi} e^x \sin(x) dx$

(Red arrow from e^x to D , Green arrow from $\sin(x)$ to P)

$$= \left[e^x x - \cos(x) \right]_0^{\pi} - \int_0^{\pi} e^x x - \cos(x) dx$$

$$= (e^{\pi} x - \cos(x)) - (e^0 x - \cos(0))$$

$$+ \int_0^{\pi} e^x \cos(x) dx$$

$$= e^{\pi} + 1 + \int_0^{\pi} e^x \cos(x) dx$$

$$J = \int_0^{\pi} e^x \cos(x) dx$$

$$= \left[e^x \sin(x) \right]_0^{\pi} - \int_0^{\pi} e^x \sin(x) dx$$

$= 0$

$$= - \int_0^{\pi} e^x \sin(x) dx$$

$$= -J$$

2)

$$I = e^{\pi} + 1 - I$$

$$2I = e^{\pi} + 1$$

$$I = \frac{e^{\pi} + 1}{2}.$$

remarque :

$$I = \int_0^{\pi} e^x \sin(x) dx$$

$$\text{or } \sin(x) = \operatorname{Im}(e^{ix})$$

$$\int_0^{\pi} e^x \cdot e^{ix} dx$$

$$= \int_0^{\pi} e^{(1+i)x} dx$$

$$= \left[\frac{1}{1+i} e^{(1+i)x} \right]_0^{\pi}$$

$$= \frac{1}{1+i} \left(e^{(1+i)\pi} - e^{(1+i)0} \right)$$

$$= \frac{1}{1+i} \left(e^{(1+i)\pi} - 1 \right)$$

$$= \frac{1-i}{2} \left(e^{(1+i)\pi} - 1 \right)$$

$$= \frac{1-i}{2} \left(\underbrace{e^{(1+i)\pi}}_{e^{\pi} \times \underbrace{e^{i\pi}}_{-1}} - 1 \right)$$

$$e^{\pi} \times \underbrace{e^{i\pi}}_{-1} = -e^{\pi}$$

$$= \overbrace{\frac{1}{2} (-e^{\pi} - 1)}^{\text{Re}}$$

$$- \frac{i}{2} (-e^{\pi} - 1)$$

$\underbrace{\hspace{15em}}_{\text{Im.}}$

donc $I = -\frac{1}{2} (-e^{\pi} - 1)$

$$= \frac{e^{\pi} + 1}{2}.$$

exercice 59:

$$1) I_0 = \int_0^{\pi/2} \cos^0(x) dx$$

$$= \int_0^{\pi/2} 1 dx$$

$$= \left[x \right]_0^{\pi/2}$$

$$= \frac{\pi}{2}.$$

$$I_1 = \int_0^{\pi/2} \cos(x) \, dx$$

$$= \left[\sin(x) \right]_0^{\pi/2}$$

$$= \sin\left(\frac{\pi}{2}\right) - \underbrace{\sin(0)}_{=0}$$

$$= 1$$

