Intégration

exercice 59 :

2) $$I_{n+2} = \int_0^{\pi/2} \cos^{n+2}(x)\,dx$$

$$= \int_0^{\pi/2} \underbrace{\cos(x)}_{P} \times \overbrace{\cos^{n+1}(x)}^{D}\,dx$$

$$= \underbrace{\Big[\sin(x)\cos^{n+1}(x)\Big]_0^{\pi/2}}_{=0}$$

$$- \int_0^{\pi/2} \sin(x) \times (n+1) \times -\sin(x) \times \cos^{n}(x)\,dx$$

$$I_{n+2} = (n+1) \int_0^{\pi/2} \sin^2(x) \cos^n(x) \, dx$$

$$= (n+1) \int_0^{\pi/2} (1 - \cos^2(x)) \cos^n(x) \, dx$$

$$= (n+1) \int_0^{\pi/2} \cos^n(x) - \cos^{n+2}(x) \, dx$$

$$= (n+1) \left[\int_0^{\pi/2} \cos^n(x) \, dx - \int_\pi^{\pi/2} \cos^{n+2}(x) \, dx \right]$$

$$= (n+1) I_n - (n+1) I_{n+2} .$$

$$I_{n+2} = (n+1) I_n - (n+1) I_{n+2}$$

$$I_{n+2} + (n+1) I_{n+2} = (n+1) I_n$$

$$(n+2) I_{n+2} = (n+1) I_n$$

$$I_{n+2} = \frac{(n+1)}{(n+2)} I_n .$$

3) $$I_{n+2} = \frac{n+1}{n+2} I_n$$

$$I_n = \frac{n-2+1}{n-2+2} I_{n-2}$$

$$I_n = \frac{n-1}{n} I_{n-2}$$

$$I_{2n} = \frac{2n-1}{2n} I_{2n-2}$$

$$= \frac{2n-1}{2n} \times \frac{2n-3}{2n-2} I_{2n-4}$$

$$= \frac{2n-1}{2n} \times \frac{2n-3}{2n-2} \times \frac{2n-5}{2n-4} I_{2n-6}$$

$$= \cdots$$

$$= \frac{2n-1}{2n} \times \frac{2n-3}{2n-2} \times \frac{2n-5}{2n-4} \times \cdots \times \frac{1}{2} I_0$$

$$= \frac{2n-1}{2n} \times \frac{2n-3}{2n-2} \times \cdots \times \frac{1}{2} \times \frac{\pi}{2}.$$

exercice 60 :

1) on pose
- $u = 2x - 1$
- $du = 2\,dx$

$\hookrightarrow dx = \dfrac{du}{2}$

- $x = 1 \Rightarrow u = 2\times 1 - 1$
$= 1$

$x = 3 \Rightarrow u = 2\times 3 - 1 = 5$

$$I = \int_1^5 \sqrt{u}\ \frac{du}{2}$$

$$= \frac{1}{2}\int_1^5 u^{1/2}\, du$$

$$= \frac{1}{2}\left[\frac{1}{\frac{1}{2}+1} u^{\frac{1}{2}+1}\right]_1^5$$

$$= \frac{1}{2} \times \frac{2}{3}\left(5^{3/2} - 1^{3/2}\right)$$

$\frac{1}{2}+1$

$5^{1/2} \times 5^{1}$

$$= \frac{1}{3}\left(5\sqrt{5} - 1\right).$$

$\underline{E9}$: $I = \displaystyle\int_1^3 \sqrt{2x-1}\, dx$

$u' \times u^n$

$$= \int_1^3 (2x-1)^{1/2}\, dx$$

$$= \frac{1}{2}\left[\frac{1}{\frac{1}{2}+1}(2x-1)^{\frac{1}{2}+1}\right]_1^3$$

$$= \frac{1}{2} \times \frac{2}{3}\left(5^{3/2} - 1^{3/2}\right).$$