Second degré

3 - Equations et Inéquations

→ $ax^2 + bx + c = 0$

$$\Delta = b^2 - 4ac$$

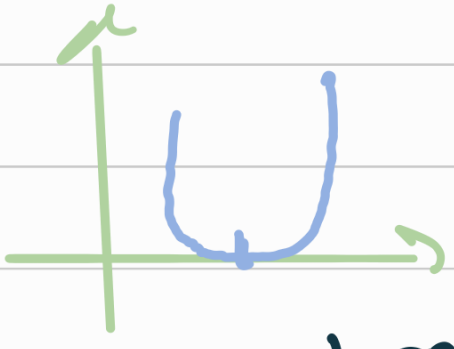
- $\Delta < 0$: pas de solution
- $\Delta = 0$: $x_0 = \frac{-b}{2a}$
- $\Delta > 0$: $x_1 = \frac{-b - \sqrt{\Delta}}{2a}$; $x_2 = \frac{-b + \sqrt{\Delta}}{2a}$

-> tableaux de signe :

■ $\Delta < 0$

x	$-\infty$	$+\infty$
signe ax^2+bx+c	Signe de a	

■ $\Delta = 0$



x	$-\infty$	x_0	$+\infty$
signe x^2+bx+c	signe de a	0	signe de a

■ $\Delta > 0$:

x	$-\infty$		x_1		x_2		$+\infty$
Signe ax^2+bx+c	signe de a		0	signe de $-a$	0	signe de a	

4 - Degrés supérieurs

■ $ax^3 + bx^2 + cx + d$.

Factoriser le polynôme / fonction

$$P(x) = x^3 - 4x^2 - 7x + 10$$

① Chercher une racine évidente

→ $-1, 1, 2, -2$

$P(1) = 1 - 4 - 7 + 10 = 0$

donc 1 racine évidente et on a :

$$P(x) = (x-1)(ax^2+bx+c)$$

② Trouver a, b, c

$$P(x) = (x-1)(ax^2+bx+c)$$

$$= ax^3 + bx^2 + cx - ax^2 - bx - c$$

$$= ax^3 + x^2(b-a) + x(c-b) - c$$

$$\begin{cases} a = 1 \\ b - a = -4 \Rightarrow b - 1 = -4 \\ \qquad\qquad \Rightarrow b = -4 + 1 = -3 \\ c - b = -7 \Rightarrow c + 3 = -7 \\ \qquad\qquad \Rightarrow c = -7 - 3 = -10 \\ -c = 10 \Rightarrow c = -10 \end{cases}$$

donc :

$$P(x) = (x-1)(x^2 - 3x - 10)$$

3 Factorise $x^2 - 3x - 10$

$$P(x) = (x-1)(x+2)(x-5)$$

■ $ax^4 + bx^2 + c = 0$

Factoriser le polynôme

$$P(x) = 25x^4 - 30x^2 + 9.$$

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on pose $X = x^2$

$$25x^4 - 30x^2 + 9 = 25X^2 - 30X + 9$$

$$\Delta = b^2 - 4ac = 0$$

Là, $X_0 = \frac{-b}{2a} = \frac{30}{50} = \frac{3}{5}$.

donc :

$$25X^2 - 30X + 9 = 25\left(X - \frac{3}{5}\right)^2.$$

$$= 25\left(x^2 - \frac{3}{5}\right)^2$$

donc : $P(x) = 25\left(x^2 - \frac{3}{5}\right)^2$

$= 5^2\left(x^2 - \frac{3}{5}\right)^2$

$= \left[5\left(x^2 - \frac{3}{5}\right)\right]^2$

$= \left(5x^2 - 3\right)^2$

Factoriser $x^4 - 5x^2 + 4$.

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on pose $X = x^2$.

$x^4 - 5x^2 + 4 = X^2 - 5X + 4$

$\Delta = b^2 - 4ac = 9 > 0$

$x_1 = \frac{-b - \sqrt{\Delta}}{2a} = 1$

$$x_2 = \frac{-b + \sqrt{\Delta}}{2a} = 4$$

$$x^2 - 5x + 4 = (x - 1)(x - 4)$$

donc :

$$x^4 - 5x^2 + 4 = (x^2 - 1)(x^2 - 4)$$

($a^2 - b^2$ et $a^2 - b^2$)

$$a^2 - b^2 = (a - b)(a + b)$$

1) $(x^2 - a) = (x - \sqrt{a})(x + \sqrt{a})$

$$= (x - 1)(x + 1)(x - 2)(x + 2)$$