

TD 2

exercice 7:

5) $\omega(x, y)$

$$= \omega(x(p, q, s), y(p, u, v))$$

$$= \omega(x(p(t), q, s(u, v)), y(p(t), u, v))$$

$$\bullet \quad \frac{\partial \omega}{\partial t} = \frac{\partial \omega}{\partial x} \times \frac{\partial x}{\partial t} +$$

$$\frac{\partial \omega}{\partial y} \times \frac{\partial y}{\partial t}$$

$$\text{or } x(p, q, s) = x(p(t), q, s(u, v))$$

$$\text{donc } \frac{\partial x}{\partial t} = \frac{\partial x}{\partial p} \times \frac{\partial p}{\partial t} +$$

$$\frac{\partial x}{\partial q} \times \frac{\partial q}{\partial t} +$$

$$\frac{\partial x}{\partial s} \times \frac{\partial s}{\partial t}$$

$$= \frac{\partial x}{\partial p} \times \frac{dp}{dt}$$

$$\text{or } y(p, u, v) = y(p(t), u, v).$$

donc :

$$\frac{\partial y}{\partial t} = \frac{\partial y}{\partial p} \times \frac{dp}{dt}.$$

done :

$$\begin{aligned}\frac{\partial w}{\partial t} &= \frac{\partial w}{\partial x} \times \frac{\partial x}{\partial \theta} \times \frac{d\theta}{dt} \\ &+ \frac{\partial w}{\partial y} \times \frac{\partial y}{\partial \theta} \times \frac{d\theta}{dt} \\ &= \left(\frac{\partial w}{\partial x} \times \frac{\partial x}{\partial \theta} + \frac{\partial w}{\partial y} \times \frac{\partial y}{\partial \theta} \right) \frac{d\theta}{dt}\end{aligned}$$

$$\bullet \quad \frac{\partial w}{\partial r} = \frac{\partial w}{\partial x} \times \frac{\partial x}{\partial r} +$$

$$\frac{\partial w}{\partial y} \times \frac{\partial y}{\partial r}$$

$$\text{or } x(p, q, s) = x(p(r), q, s(u, v))$$

done :

$$\frac{\partial x}{\partial v} = \frac{\partial x}{\partial p} \times \frac{\partial p}{\partial v} = 0$$

$$+ \frac{\partial x}{\partial q} \times \frac{\partial q}{\partial v} = 0$$

$$+ \frac{\partial x}{\partial s} \times \frac{\partial s}{\partial v}$$

$$\text{d'au} : \frac{\partial x}{\partial v} = \frac{\partial x}{\partial p} \times \frac{\partial s}{\partial v}$$

$$\alpha \quad y(p, u, v) = y(p(t), u, v)$$

$$\frac{\partial y}{\partial v} = \frac{\partial y}{\partial p} \times \frac{\partial p}{\partial v} + \frac{\partial y}{\partial u} \times \frac{\partial u}{\partial v} + \frac{\partial y}{\partial v} \times \frac{\partial v}{\partial v} = 1$$

$$= \frac{\partial y}{\partial v}$$

done :

$$\frac{\partial w}{\partial v} = \frac{\partial w}{\partial x} \times \frac{\partial x}{\partial s} \times \frac{\partial s}{\partial v} + \frac{\partial w}{\partial y} \times \frac{\partial y}{\partial v}$$

$$6) \quad z = f(x, y) \\ = f(x(r, s), y(r, s))$$

$$\frac{\partial z}{\partial s} = \frac{\partial f}{\partial x} \times \frac{\partial x}{\partial s} + \frac{\partial f}{\partial y} \times \frac{\partial y}{\partial s}$$

$$= \frac{\partial f}{\partial x} \times 2s + \frac{\partial f}{\partial y} \times 2r$$

$$\frac{\partial}{\partial r} \left(\frac{\partial f}{\partial s} \right) = \frac{\partial}{\partial r} \left(\frac{\partial f}{\partial x} \times 2s \right) \\ + \frac{\partial}{\partial r} \left(\frac{\partial f}{\partial y} \times 2r \right)$$

$$\bullet \frac{\partial}{\partial r} \left(\frac{\partial f}{\partial x} \times 2s \right)$$

$$= 2s \times \frac{\partial}{\partial r} \left(\frac{\partial f}{\partial x} \right)$$

$$= 2s \times \left[\frac{\partial^2 f}{\partial x^2} \times \frac{\partial x}{\partial r} + \frac{\partial^2 f}{\partial y \partial x} \frac{\partial y}{\partial r} \right]$$

$$= 2s \times \left[\frac{\partial^2 f}{\partial x^2} \times 2r + \frac{\partial^2 f}{\partial y \partial x} \times 2s \right]$$

$$= 4sr \frac{\partial^2 f}{\partial x^2} + 4s^2 \frac{\partial^2 f}{\partial y \partial x}$$

$$\bullet \frac{\partial}{\partial r} \left(\frac{\partial f}{\partial y} \times 2r \right) \rightarrow (u \times v)'$$

$$= 2r \times \left[\frac{\partial^2 f}{\partial x \partial y} \times \frac{\partial x}{\partial r} + \frac{\partial^2 f}{\partial y^2} \times \frac{\partial y}{\partial r} \right]$$

$$+ \frac{\partial f}{\partial y} \times 2$$

$$= 2r \times \left[\frac{\partial^2 f}{\partial x \partial y} \times 2r + \frac{\partial^2 f}{\partial y^2} \times 2r \right]$$

$$+ 2 \frac{\partial f}{\partial y}$$

$$= 4r^2 \frac{\partial^2 f}{\partial x \partial y} + 4r^2 \frac{\partial^2 f}{\partial y^2} + 2 \frac{\partial f}{\partial y}$$

donc :

$$\begin{aligned}\frac{\partial^2 z}{\partial x \partial y} &= 4sr \left[\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \right] \\ &\quad + 4s^2 \frac{\partial^2 f}{\partial y \partial x} + 4r^2 \frac{\partial^2 f}{\partial x \partial y} \\ &\quad + 2 \frac{\partial f}{\partial x}\end{aligned}$$

$$= 4sr \left[\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \right]$$

$$+ 4(s^2 + r^2) \frac{\partial^2 f}{\partial x \partial y} + 2 \frac{\partial f}{\partial y}$$

exercice 10 :

1) $g(x, y) = f \circ ((x, y) \mapsto (y, x))$

on pose $u = y$, $v = x$

$$g(x, y) = f(u, v)$$

$$\bullet \frac{\partial g}{\partial x} = \frac{\partial f}{\partial u} \times \frac{\partial u}{\partial x} + \frac{\partial f}{\partial v} \times \frac{\partial v}{\partial x}$$

$$= \frac{\partial f}{\partial u} \times 0 + \frac{\partial f}{\partial v} \times 1$$

$$= \frac{\partial f}{\partial v}$$

$$\cdot \frac{\partial g}{\partial y} = \frac{\partial f}{\partial u} \times \frac{\partial u}{\partial y} + \frac{\partial f}{\partial v} \times \frac{\partial v}{\partial y}$$

$$= \frac{\partial f}{\partial u} \times 1 + \frac{\partial f}{\partial v} \times 0$$

$$= \frac{\partial f}{\partial u}$$

$$u = y \quad v = f(x, x)$$

$$= f(x, x)$$

$$s = x, t = x$$