$$x^3 + y^3 = 6xy.$$

a)

$$\frac{d}{dx}\left(x^3 + y^3\right) = \frac{d}{dx}(6xy)$$

$y = y(x)$

$$3x^2 + 3y^2\frac{dy}{dx}$$

$$= 6y + 6x\frac{dy}{dx}.$$

$$3x^2 + 3y^2y' = 6y + 6xy'.$$

$$x^2 + y^2y' = 2y + 2xy'.$$

$$y^2y' - 2xy' = 2y - x^2$$

$$y'(y^2-2x) = 2y - x^2$$

$$\boxed{y' = \frac{2y-x^2}{y^2-2x}}$$

5) $f(x,\phi(x)) = x^2 + y^3 - 6xy$.

$$y = f'(3,\phi(3))(x-3) + f(3,\phi(3))$$

$$y'(x) = \frac{2y-x^2}{y^2-2x}$$

$$y'(3) = \frac{2\times 3 - 3^2}{3^2 - 2\times 3} = -1$$

donc $y = -1x(x-3) + 3$

$y = -x + 3 + 3$

$y = -x + 6$

$\Rightarrow x + y = 6.$

c) $y' = 0$

$\Leftrightarrow \dfrac{2y - x^2}{y^2 - 2x} = 0$

$\Leftrightarrow 2y - x^2 = 0$

$\Leftrightarrow y = \dfrac{x^2}{2}$

d'où :

$$x^3 + y^3 - 6xy = 0$$

$$x^3 + \frac{x^6}{2^3} - 6x \times \frac{x^2}{2} = 0$$

$$x^3 + \frac{x^6}{2^3} - 3x^3 = 0$$

$$-2x^3 + \frac{x^6}{2^3} = 0$$

$$x^3\left(-2 + \frac{x^3}{2^3}\right) = 0$$

$x = 0$ ou $\dfrac{x^3}{2^3} = 2$

$$x^3 = 2^4$$

$$x = 2^{\frac{4}{3}}$$

$$y = \frac{x^2}{2} = \frac{\left(2^{\frac{4}{3}}\right)^2}{2}$$

$$= 2^{\frac{8}{3} - 1}$$

$$= 2^{5/3}$$

- $x^4 + y^4 = 16.$

$$\frac{d}{dx}\left(x^4 + y^4\right) = \frac{d}{dx}(16)$$

$$4x^3 + 4y^3 y' = 0$$

$$y' = -\frac{4x^3}{4y^3} = -\frac{x^3}{y^3}$$

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$$y'' = \frac{d}{dx} y'$$

$$= \frac{d}{dx}\left(\frac{-x^3}{y^3}\right)$$

$$y'' = \frac{-3x^2y^3 - (-x^3)\times 3y^2y'}{y^6}$$

$$= \frac{-3x^2y^3 + x^3 3y^2 y'}{y^6}$$

$$= \frac{-3x^2y^3 + x^3 3y^2 \times -\frac{x^3}{y^3}}{y^6}$$

$$= \frac{-3x^2y^3 - \frac{3x^6y^2}{y^3}}{y^6}$$

$$= \frac{-3x^2y^3 - \frac{3x^6}{y}}{y^6}$$

$$= \frac{\frac{-3x^2y^4 - 3x^6}{y}}{y^6}$$

$$= \frac{-3x^2y^4 - 3x^6}{y^7}$$

$$= 16$$

$$= \frac{-3x^2(y^4 + x^4)}{y^7}$$

$$= \frac{-3 \times 16\, x^2}{y^7}$$

$$= -48\,\frac{x^2}{y^7}.$$

- $f(x,y) = 0$

⚠ $\frac{\partial f}{\partial y} \neq 0$ → y variable

$$\Rightarrow \frac{dy}{dx} = - \frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}}$$

# Implicit functions theorem

$\rightarrow f(a, b) = 0$

$\rightarrow \frac{\partial f}{\partial y}(a, b) \neq 0$

$\hookrightarrow f(x, y) = 0 \Leftrightarrow y = \phi(x)$

<u>et</u>

$$y' = \phi'(x) = -\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}}.$$