

Worksheet 4

exercice 2:

$$\textcircled{\mathbb{R}} \quad y = f'(a)(x-a) + f(a)$$

$$\textcircled{\mathbb{R}^2} \quad z = f(a, b) + \frac{\partial f}{\partial x}(a, b)(x-a) + \frac{\partial f}{\partial y}(a, b)(y-b)$$

$$\textcircled{\mathbb{R}^3} : f(a, b, c) + \frac{\partial f}{\partial x}(a, b, c)(x-a) + \frac{\partial f}{\partial y}(a, b, c)(y-b) + \frac{\partial f}{\partial z}(a, b, c)(z-c) = 0.$$

$$1) f(x, y, z) = \sqrt{xyz}$$

$$\frac{\partial f}{\partial x}(x, y, z) = \frac{yz}{2\sqrt{xyz}}$$

$$\frac{\partial f}{\partial y}(x, y, z) = \frac{xz}{2\sqrt{xyz}}$$

$$\frac{\partial f}{\partial z}(x, y, z) = \frac{xy}{2\sqrt{xyz}}$$

$$\bullet f(100, 36, 1) = \sqrt{36 \times 100}$$

$$= 6 \times 10 = 60$$

$$\frac{\partial f}{\partial x}(100, 36, 1) = \frac{36}{2 \times 60} = \frac{18}{60} = \frac{3}{10}$$

$$\cdot \frac{\partial f}{\partial y}(100, 36, 1) = \frac{100}{2 \times 60} = \frac{50}{60} = \frac{5}{6}$$

$$\begin{aligned} \cdot \frac{\partial f}{\partial z}(100, 36, 1) &= \frac{100 \times 36}{2 \times 60} = 50 \times \frac{6}{10} \\ &= 5 \times 6 \\ &= 30 \end{aligned}$$

$$\begin{aligned} T: 60 + \frac{3}{10}(x - 100) + \frac{5}{6}(y - 36) \\ + 30(z - 1) = 0 \end{aligned}$$

$$\begin{aligned} T: 60 + \frac{3}{10}x - 30 + \frac{5}{6}y - 30 \\ + 30z - 30 = 0 \end{aligned}$$

$$T: \quad \frac{3}{10}x + \frac{5}{6}y + 30z = 30$$

$$2) \quad f(x, y, z) = f(x_0, y_0, z_0)$$

$$+ \frac{\partial f}{\partial x}(x_0, y_0, z_0)(x - x_0)$$

$$+ \frac{\partial f}{\partial y}(x_0, y_0, z_0)(y - y_0)$$

$$+ \frac{\partial f}{\partial z}(x_0, y_0, z_0)(z - z_0)$$

$$+ \frac{1}{2} \left[\frac{\partial^2 f}{\partial x^2}(x_0, y_0, z_0)(x - x_0)^2 \right.$$

$$+ 2 \frac{\partial^2 f}{\partial x \partial y}(x_0, y_0, z_0)(x - x_0)(y - y_0)$$

$$+ 2 \frac{\partial^2 f}{\partial x \partial z} (x_0, y_0, z_0) (x - x_0) (z - z_0)$$

$$+ \frac{\partial^2 f}{\partial y^2} (x_0, y_0, z_0) (y - y_0)^2$$

$$+ 2 \frac{\partial^2 f}{\partial y \partial z} (x_0, y_0, z_0) (y - y_0) (z - z_0)$$

$$+ \frac{\partial^2 f}{\partial x^2} (x_0, y_0, z_0) (x - x_0)^2$$

$$\frac{\partial f}{\partial x^2} (x, z) = \frac{y z}{2} \times y^{\frac{1}{2}} z^{\frac{1}{2}} x^{-\frac{3}{2}} - \frac{1}{2} x^{-\frac{3}{2}}$$

$$= \frac{-\sqrt{y} \sqrt{z}}{4 x^{3/2}}$$

$$\frac{\partial^2 f(x,y,z)}{\partial y^2} = -\frac{\sqrt{x} \sqrt{z}}{4 y^{3/2}}$$

$$\frac{\partial^2 f(x,y,z)}{\partial z^2} = -\frac{\sqrt{x} \sqrt{y}}{4 z^{3/2}}$$

$$u(y) = yz \Rightarrow u'(y) = z$$

$$\begin{aligned} v(y) &= 2\sqrt{xyz} \Rightarrow v'(y) = 2 \times \sqrt{z} \times \sqrt{x} \\ &= 2\sqrt{x} \sqrt{y} \sqrt{z} \times \frac{1}{2\sqrt{y}} \end{aligned}$$

$$\frac{\partial^2 f(x,y,z)}{\partial y^2} = \frac{z \times 2\sqrt{x} \sqrt{y} \sqrt{z} - yz \times 2 \times \sqrt{x} \times \sqrt{z} \times \frac{1}{2\sqrt{y}}}{(2\sqrt{xyz})^2}$$

$$= \frac{2\sqrt{x}\sqrt{y}z^{3/2} - \sqrt{y}z^{3/2}\sqrt{x}}{4xyz}$$

$$= \frac{\sqrt{x}\sqrt{y}z^{3/2}}{4xyz}$$

$$= \frac{\sqrt{z}}{4\sqrt{x}\sqrt{y}}$$

$$\frac{\partial^2 f}{\partial y \partial z} = \frac{\sqrt{x}}{4\sqrt{y}\sqrt{z}}$$

$$\frac{\partial^2 f}{\partial x \partial z} = \frac{\sqrt{y}}{4\sqrt{x}\sqrt{z}}.$$

$$\sqrt{1091 \times 36,1 \times 0,999}$$

$$= 60 + \frac{3}{10} (0,1) + \frac{5}{6} (0,1) + 30 (-0,001) + \frac{1}{2} \left[-\frac{3}{2000} (0,1)^2 \right.$$

$$+ 2 \times \frac{1}{240} \times (0,1)(0,1) + 2 \times \frac{3}{20} \times (0,1) \times (-0,001)$$

$$+ -\frac{5}{432} \times (0,1)^2 + 2 \times \frac{5}{12} (0,1)(-0,001) + (-15) \times (-0,001)^2 \Big]$$

$$\approx 60,083245463.$$