Fiche 7

exercice 1 :

a) $\begin{cases} 5x' - 8x + 9y = 0 \\ 5y' - 6x + 13y = 0 \end{cases}$

$\Leftrightarrow \begin{cases} x' = \frac{8}{5}x - \frac{9}{5}y \\ y' = \frac{6}{5}x - \frac{13}{5}y \end{cases}$

- point stationnaire : $(0;0)$.

on pose $A = \begin{pmatrix} 8/5 & -9/5 \\ 6/5 & -13/5 \end{pmatrix}$

telle que : $Y' = AY$, $Y\begin{pmatrix} x \\ y \end{pmatrix}$.

on pose $B = \begin{pmatrix} 8 & -9 \\ 6 & -13 \end{pmatrix}$

$$P_B(X) = \begin{vmatrix} 8-X & -9 \\ 6 & -13-X \end{vmatrix}$$

$$= (8-X)(-13-X) + 54$$

$$= -104 - 8X + 13X + X^2 + 54$$

$$= X^2 + 5X - 50$$

$$\Delta = b^2 - 4ac = 25 + 200$$

$$= 225$$

$$x_1 = \frac{-b - \sqrt{\Delta}}{2a}$$

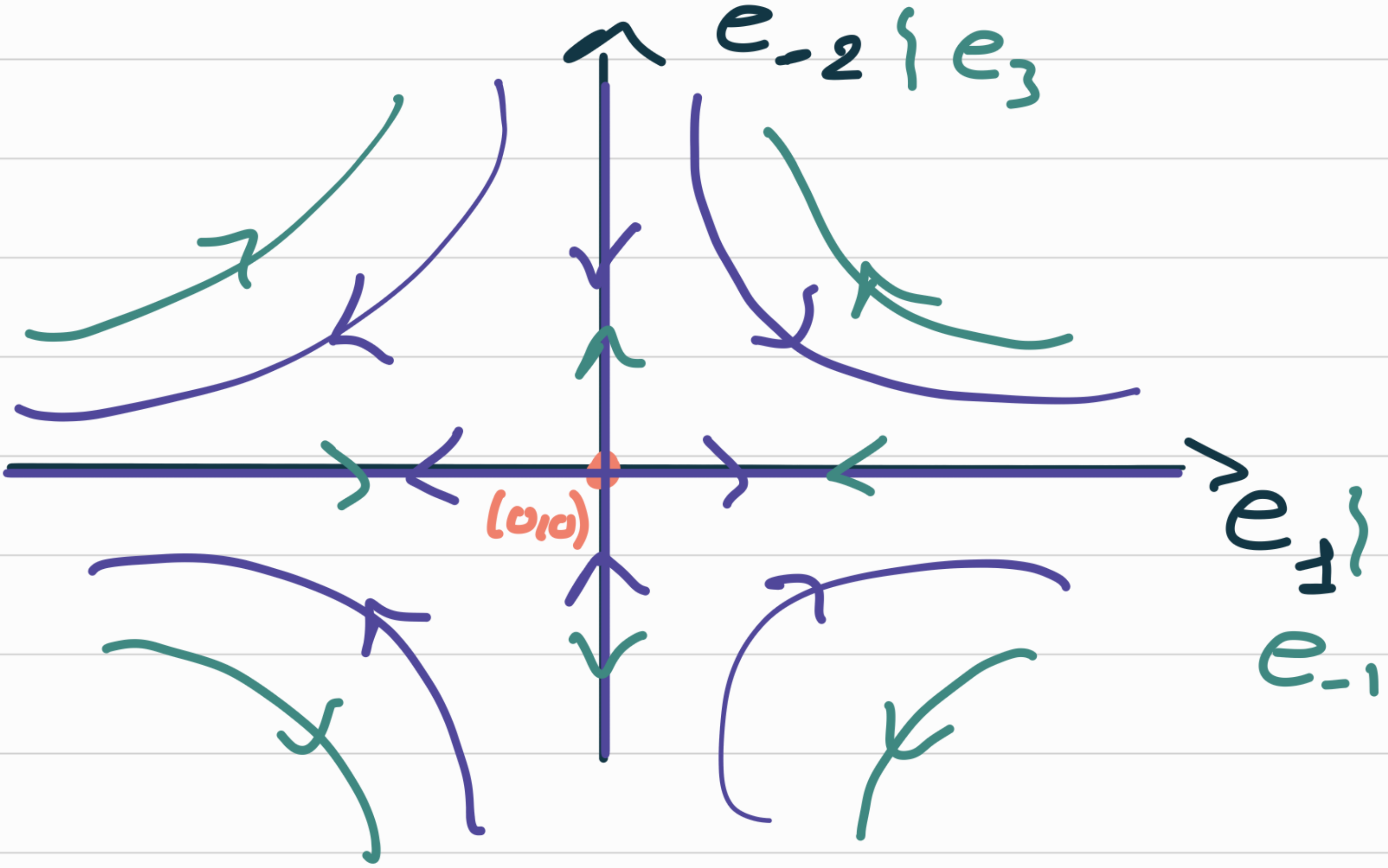
$$= \frac{-5 - 15}{2} = -10$$

$$x_2 = \frac{-5 + 15}{2} = 5$$

donc $P_B(X) = (X+10)(X-5)$

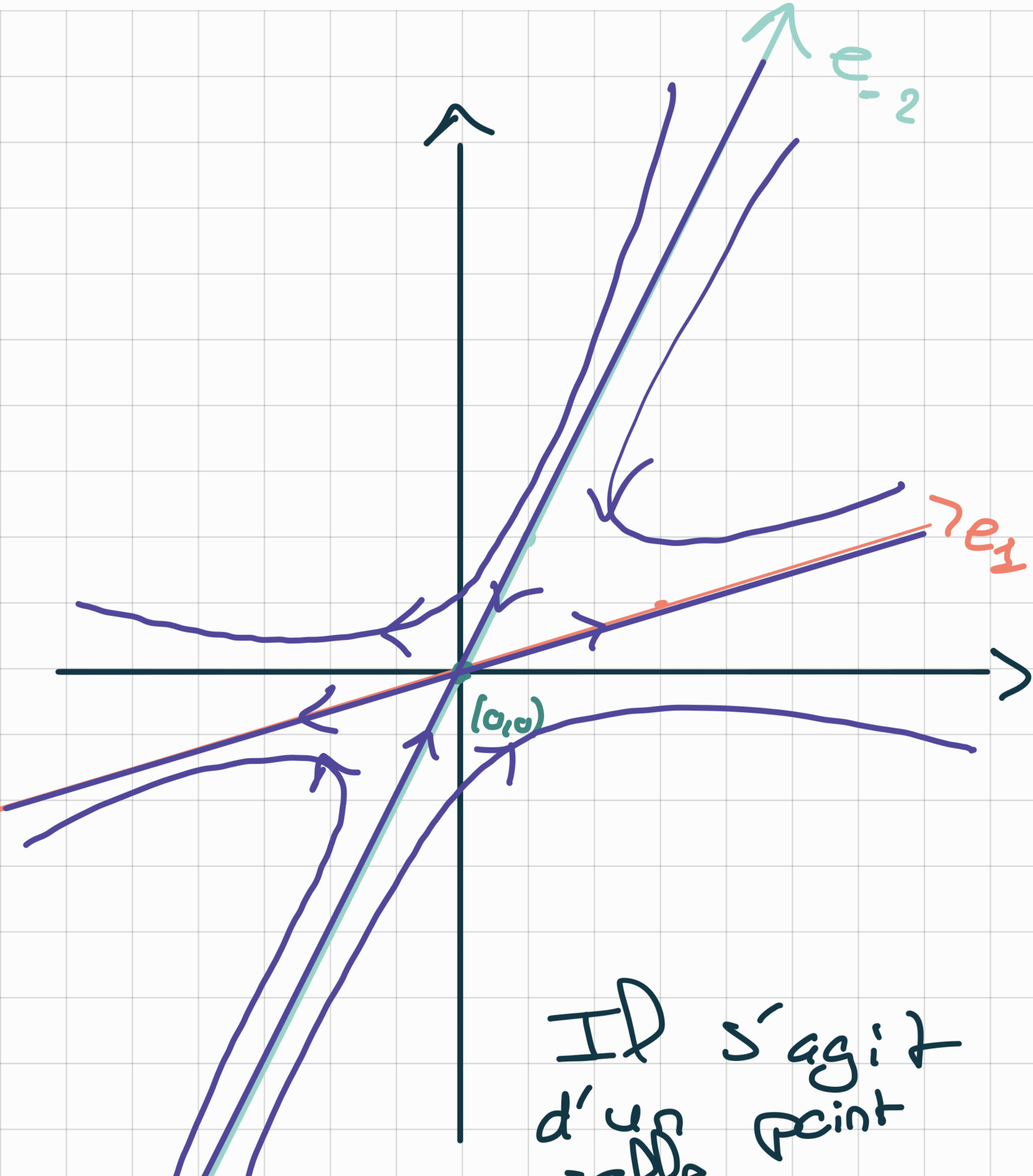
et les valeurs de propres de A sont -2 et 1.

Donc A est diagonalisable avec $-2 < 0 < 1$.



$$e_1 = \mathbb{R} \cdot \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$$e_{-2} = \mathbb{R} \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$



Il s'agit d'un point selle.

b) $\begin{cases} 3x' + 7x - 5y = 0 \\ 3y' + 2x + 5y = 0 \end{cases}$

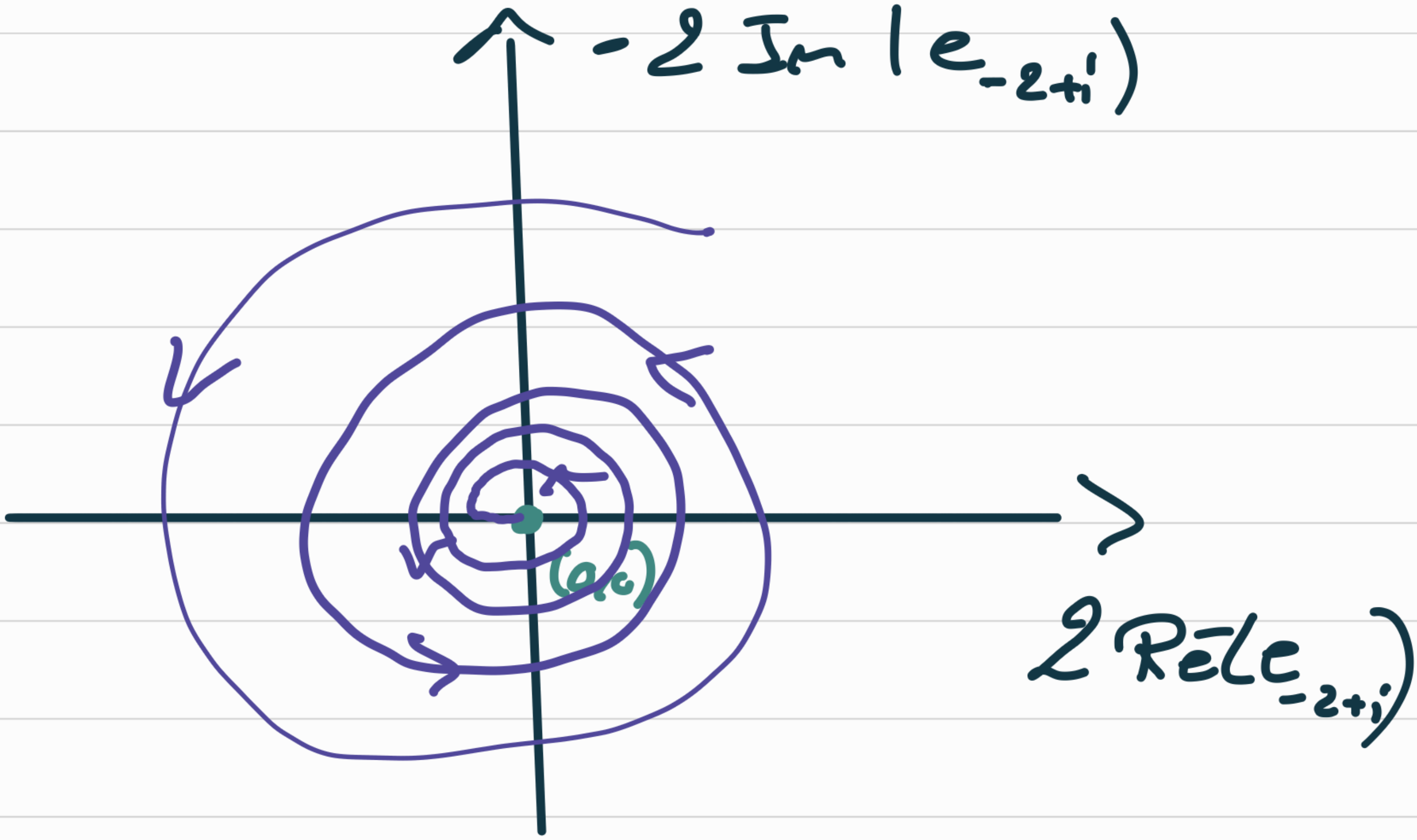
$\Leftrightarrow \begin{cases} x' = -\frac{7}{3}x + \frac{5}{3}y \\ y' = -\frac{2}{3}x - \frac{5}{3}y \end{cases}$

on pose $A = \frac{1}{3}\begin{pmatrix} -7 & 5 \\ -2 & -5 \end{pmatrix}$

telle que $Y' = AY$, $Y\begin{pmatrix} x \\ y \end{pmatrix}$

. $P_A(X) = X^2 + 4X + 5$

Les valeurs propres : $-2 \pm i$.



Calculons e_{-2+i}.

$$e_{-2+i} = \ker\left(A - (-2+i) I_2\right)$$

$$= \ker \begin{pmatrix} -\frac{1}{3} - i & \frac{5}{3} \\ -\frac{2}{3} & \frac{1}{3} - i \end{pmatrix}$$

$$= \ker \frac{1}{3} \begin{pmatrix} -1-3i & 5 \\ -2 & 1-3i \end{pmatrix}$$

$$\sim \ker \begin{pmatrix} -2 & 1-3i \\ -2-6i & 10 \end{pmatrix}$$

$$\sim \ker \begin{pmatrix} -2 & 1-3i \\ -6i & 9+3i \end{pmatrix}$$

$$\sim \ker \begin{pmatrix} -2 & 1-3i \\ 0 & 0 \end{pmatrix}$$

$L_2 - 3iL_1$

$$\iff -2x + (1-3i)y = 0$$

$$\iff x = \left(\frac{1}{2} - \frac{3i}{2}\right)y$$

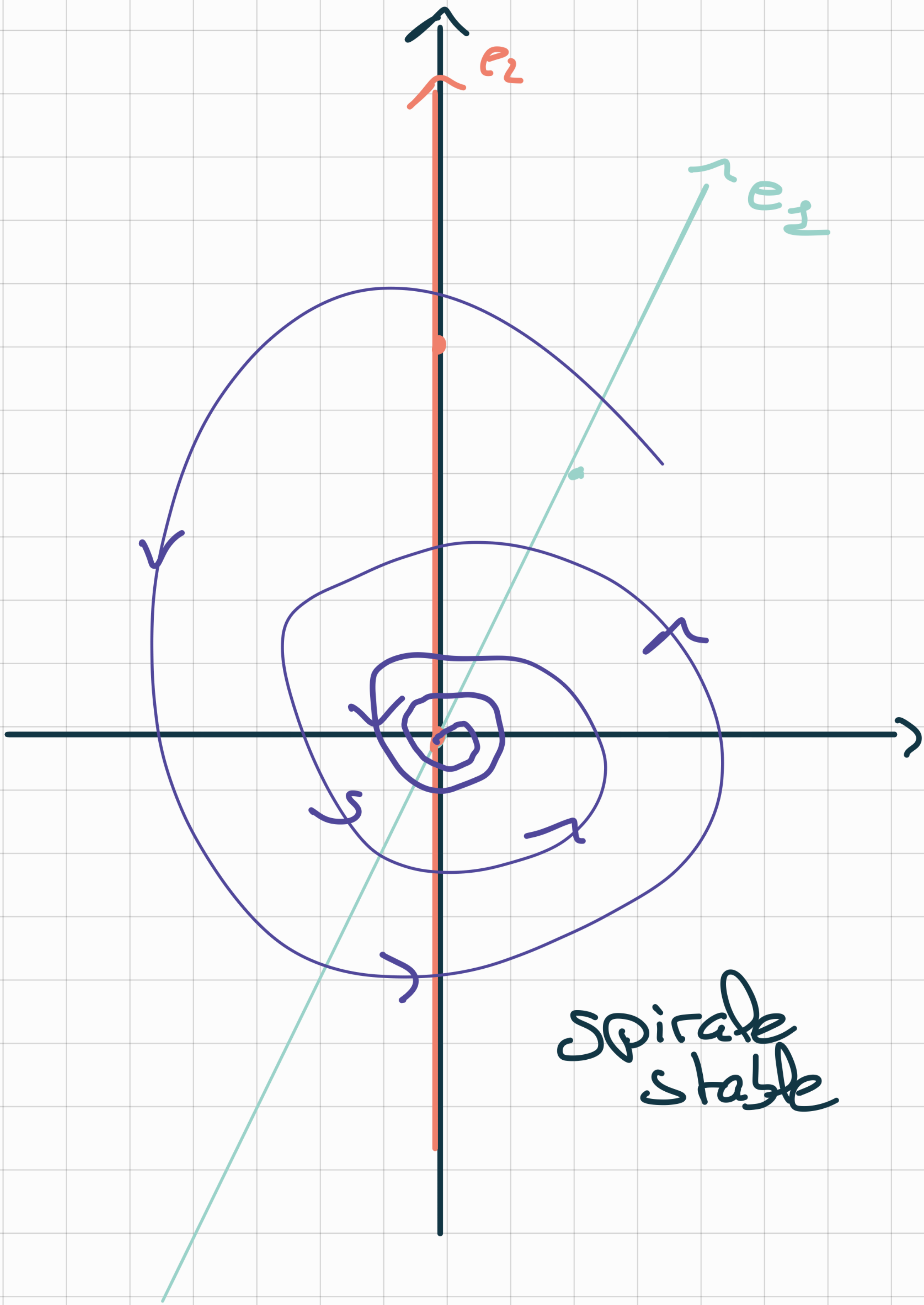
$$E_{-2+i} = \mathbb{C} \cdot \begin{pmatrix} \frac{1}{2} - \frac{3}{2}i \\ 1 \end{pmatrix} = \mathbb{C}\begin{pmatrix} 1-3i \\ 2 \end{pmatrix}$$

$$e_{-2+i} = \mathbb{R}\cdot\begin{pmatrix}1\\2\end{pmatrix} + i\mathbb{R}\begin{pmatrix}-3\\0\end{pmatrix}$$

$$\begin{pmatrix}5\\1\end{pmatrix} \qquad \begin{pmatrix}0\\3\end{pmatrix}$$

$$e_1 = 2\bar{Re}\left(e_{-2+i}\right) = \begin{pmatrix}2\\4\end{pmatrix} \begin{pmatrix}10\\2\end{pmatrix}$$

$$e_2 = -2\,\mathrm{Im}\left(e_{2+i}\right) = \begin{pmatrix}6\\0\end{pmatrix} \begin{pmatrix}0\\-6\end{pmatrix}$$



e_2
e_1
spirale stable

exercice 2 :

b) $\begin{cases} x' = -2x + 2y - 2 & f_v \\ y' = \frac{x}{2} - 2y + \frac{7}{2} & f_h \end{cases}$

on a : $\begin{cases} -2x + 2y - 2 = 0 \\ \frac{x}{2} - 2y + \frac{7}{2} = 0 \end{cases}$

$\Leftrightarrow \begin{cases} x = 1 \\ y = 2 \end{cases}$

donc il y a un point stationnaire en $(1 ; 2)$.

on pose :

$$\begin{cases} \tilde{x} = x - 1 \\ \tilde{y} = y - 2 \end{cases}$$

$\bullet\ x' + 2x - 2y + 2$

$= \tilde{x}' + 2(\tilde{x}+1) - 2(\tilde{y}+2) + 2$

$= \boxed{\tilde{x}' + 2\tilde{x} - 2\tilde{y}} = 0$

$\bullet\ 2y' - x + 4y - 7$

$= 2\tilde{y}' - (\tilde{x}+1) + 4(\tilde{y}+2) - 7$

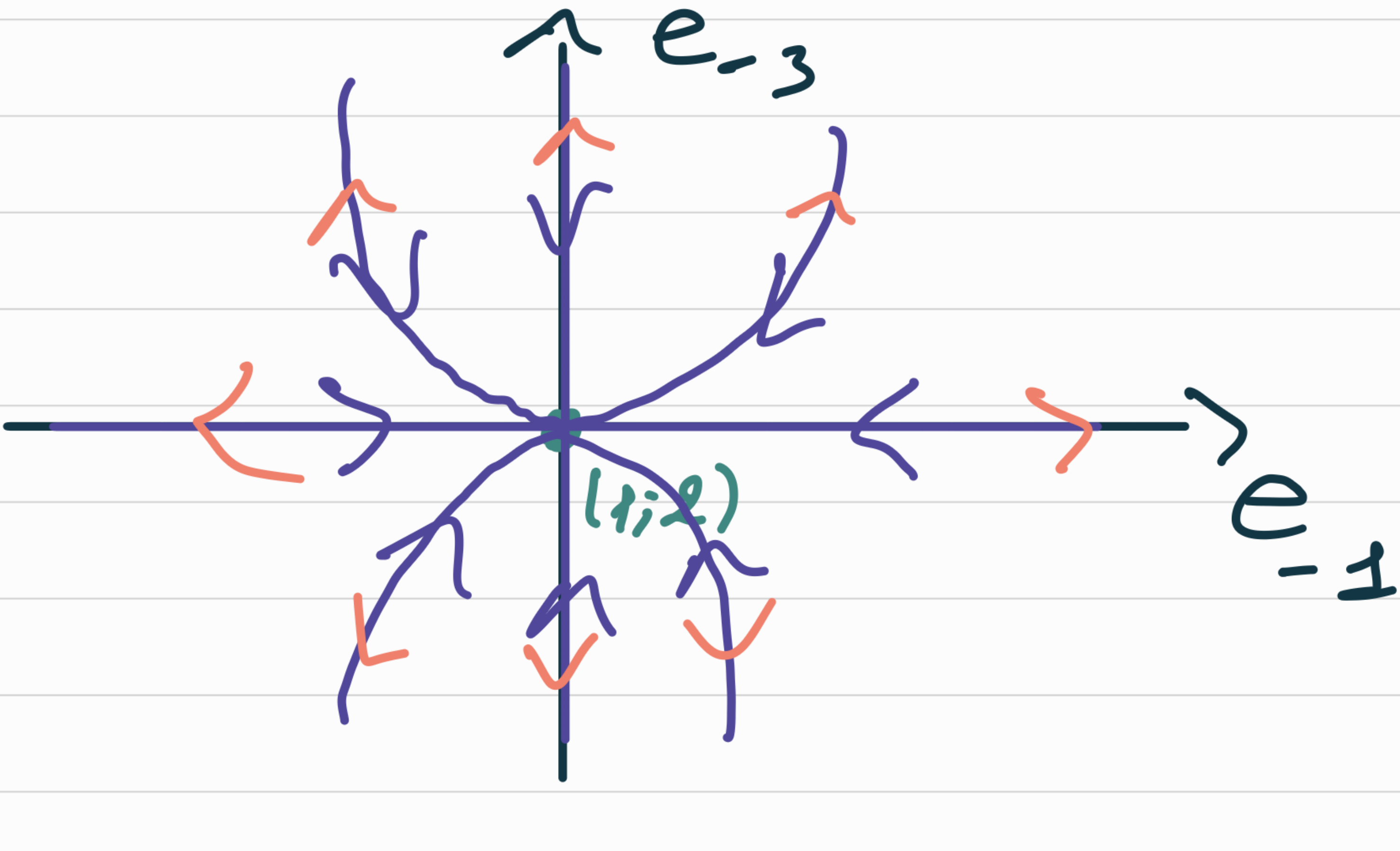
$= \boxed{2\tilde{y}' - \tilde{x} + 4\tilde{y}.} = 0$

on pose $A = \begin{pmatrix} -2 & 2 \\ \frac{1}{2} & -2 \end{pmatrix}$

telle $\tilde{Y}' = A\tilde{Y}$, $\tilde{Y} = \begin{pmatrix} \tilde{x} \\ \tilde{y} \end{pmatrix}$

$\Rightarrow$ valeurs propres de A sont -1 et -3.

Donc A est diagonalisable avec $-3 < -1 < 0$



$$e_{-1} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$e_{-3} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$