- **Cas 2** : les valeurs propres sont réelles et A n'est pas diagonalisable.

$$A \overset{\text{semblable}}{\sim} \begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix}$$

on note λ la valeur propre et (e_1, e_2) une base telle que :

$$\begin{cases} A e_1 = \lambda e_1 \\ A e_2 = e_1 + \lambda e_2 \end{cases}$$

les solutions sont de la forme :

$$\begin{cases} \tilde{x}(t) = (C_1 + t C_2) e^{\lambda t} \\ \tilde{y}(t) = C_2 e^{\lambda t} \end{cases}$$

$\rightarrow$ $\boxed{C_2 = 0}$

* $C_1 = C_2 = 0$

$\mathcal{E}_1$ courbe intégrale $\{(0,0)\}$.

* $C_2 = 0,\ C_1 > 0$

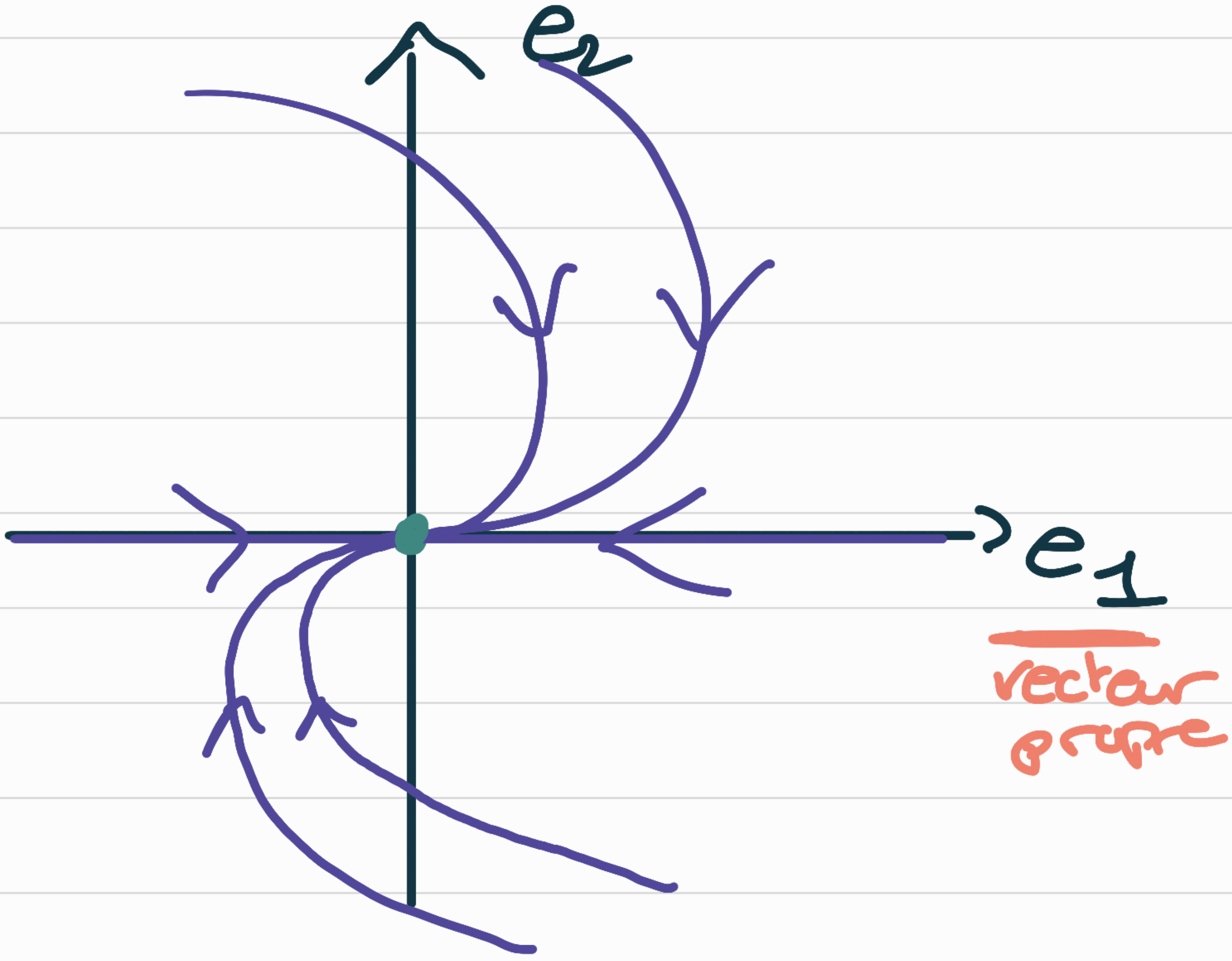
$\mathcal{E}_1$ courbe intégrale $\{(\tilde{x},0) : \tilde{x}>0\}$

* $C_2 = 0,\ C_1 < 0$

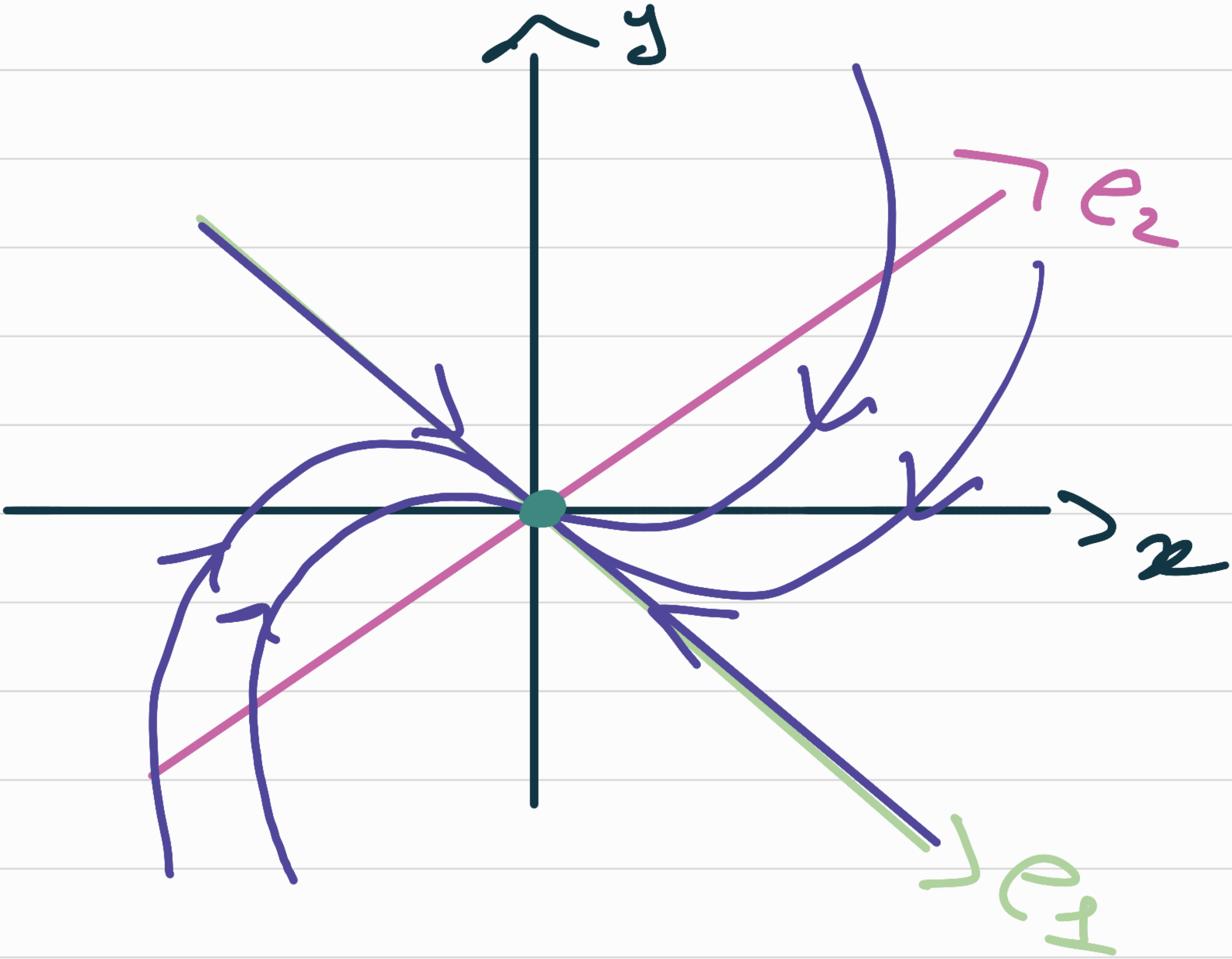
$\mathcal{E}_1$ courbe intégrale $\{(\tilde{x},0) : \tilde{x}<0\}$

→ $\boxed{c_2 \neq 0}$

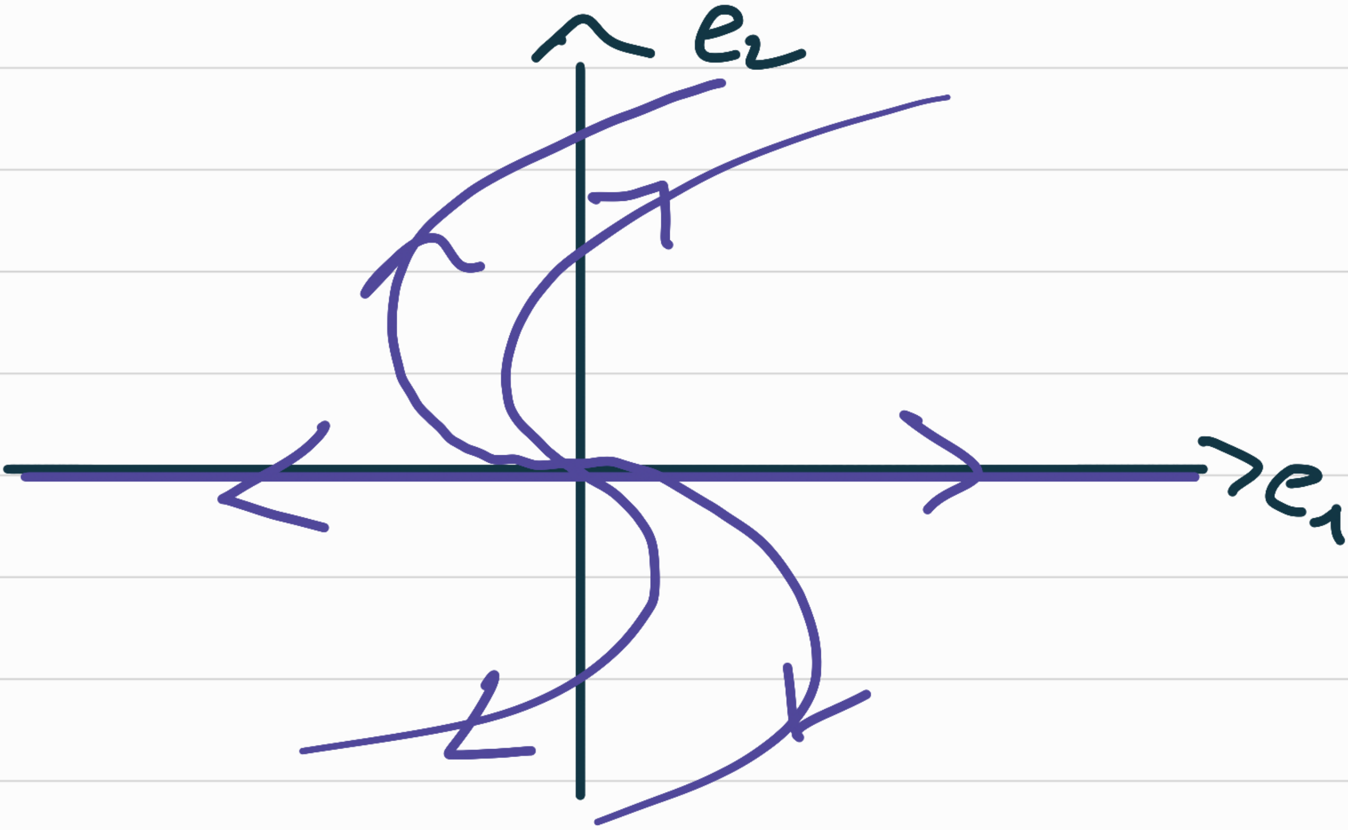
* $\lambda < 0$



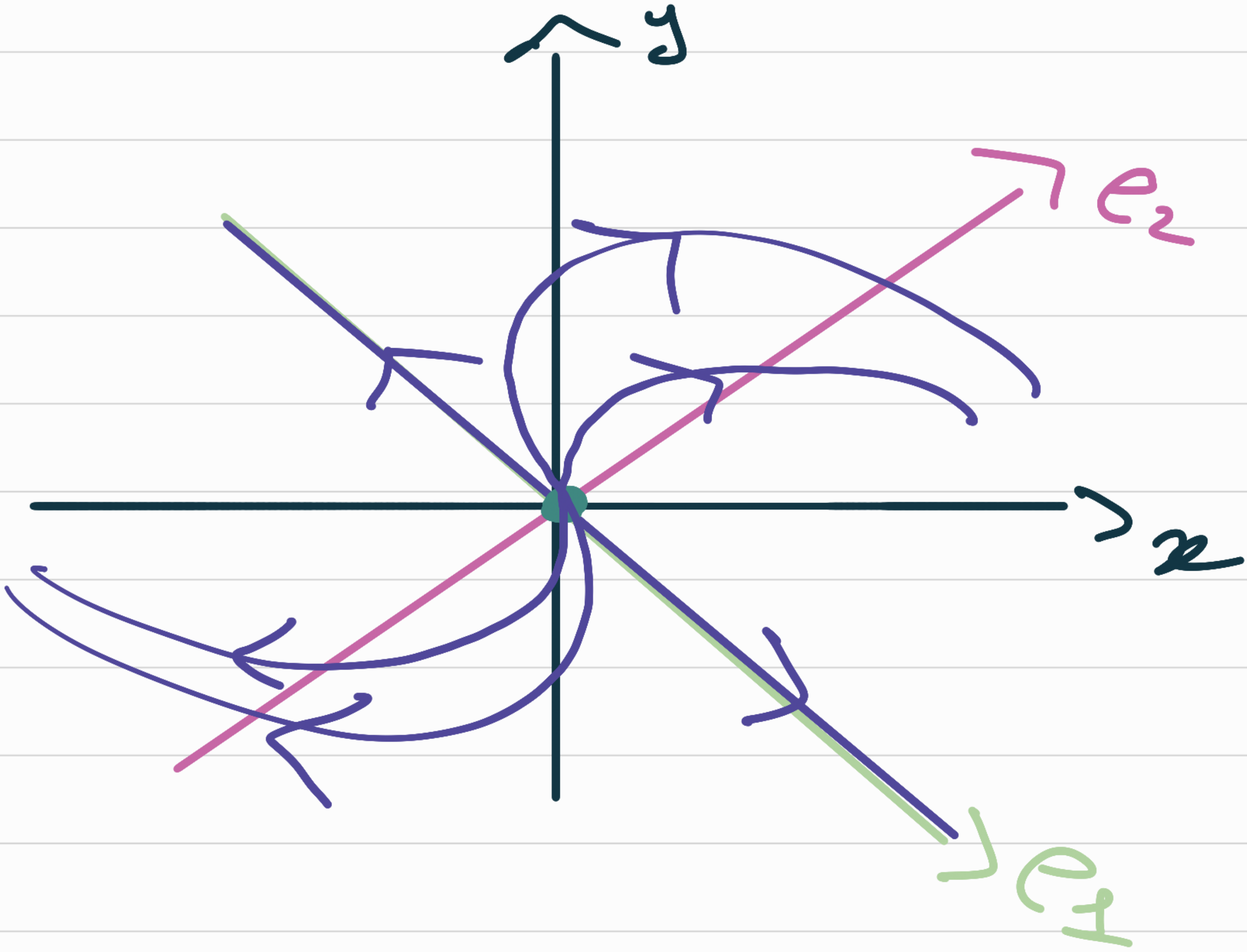
noeud dégénéré attractif.



* $\lambda > 0$



nœud dégénéré répulsif.



• **Cas 3** : les valeurs propres sont complexes et A diagonalisable dans $\mathbb{C}$.

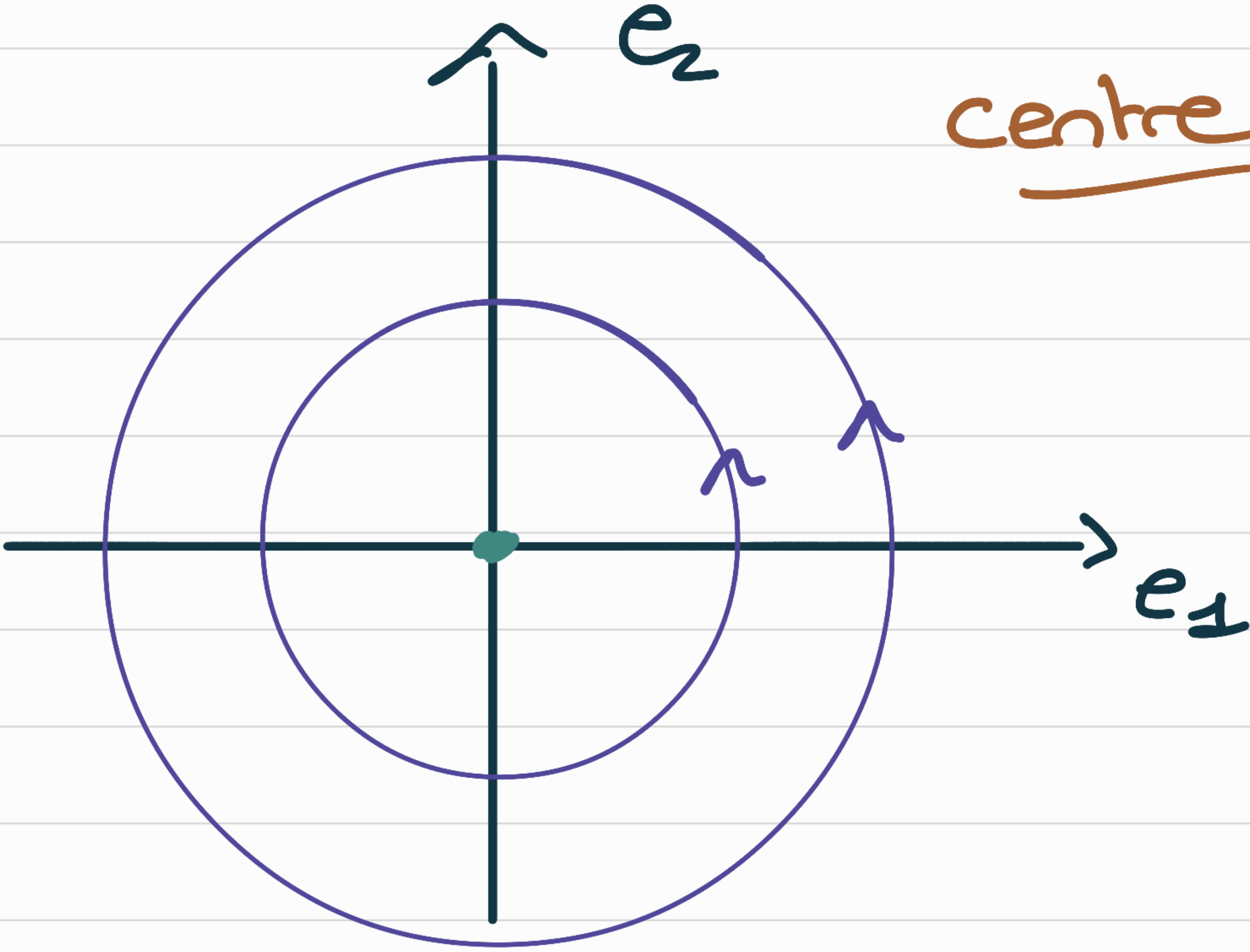
$$\begin{cases} \lambda = \alpha + i\beta \\ \mu = \alpha - i\beta \end{cases} , \quad \boxed{\beta > 0}$$

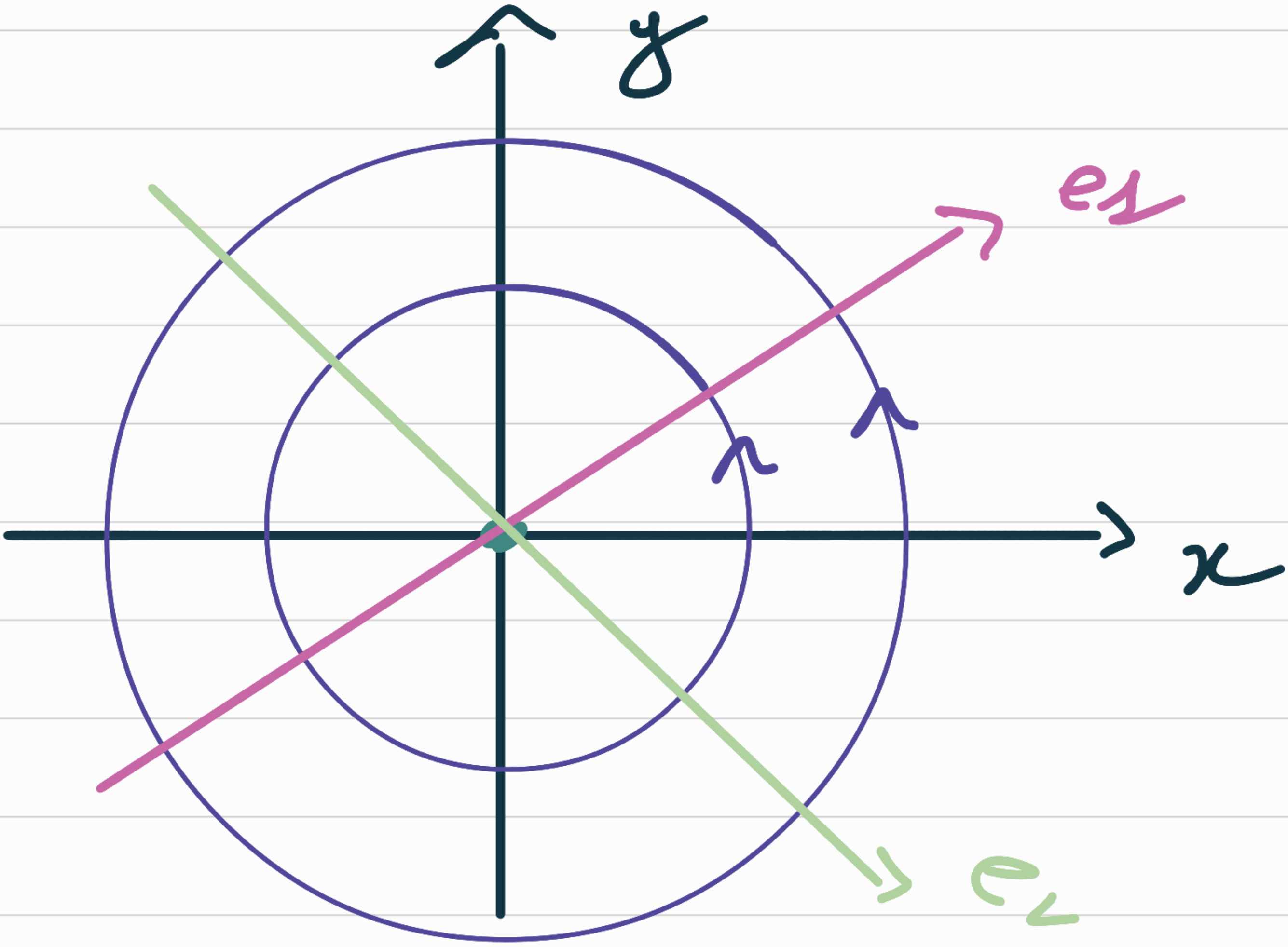
A semblable à $\begin{pmatrix} \alpha & -\beta \\ \beta & \alpha \end{pmatrix}$

$$\boxed{\begin{aligned} e_1 &= \ell\,\mathrm{Re}(e_\lambda) \\ e_2 &= -\ell\,\mathrm{Im}(e_\lambda) \end{aligned}}$$

$\rightarrow \boxed{\alpha = 0}$

centre

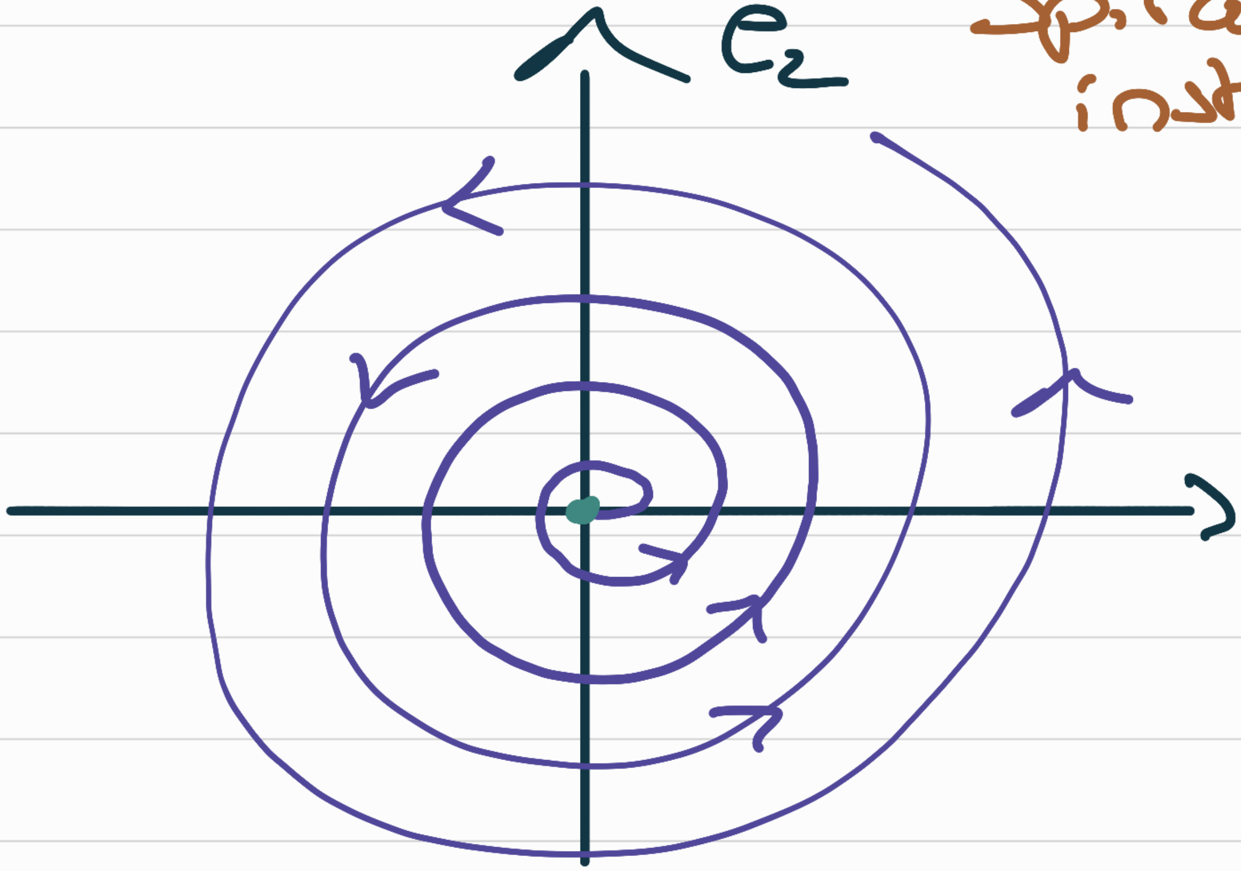


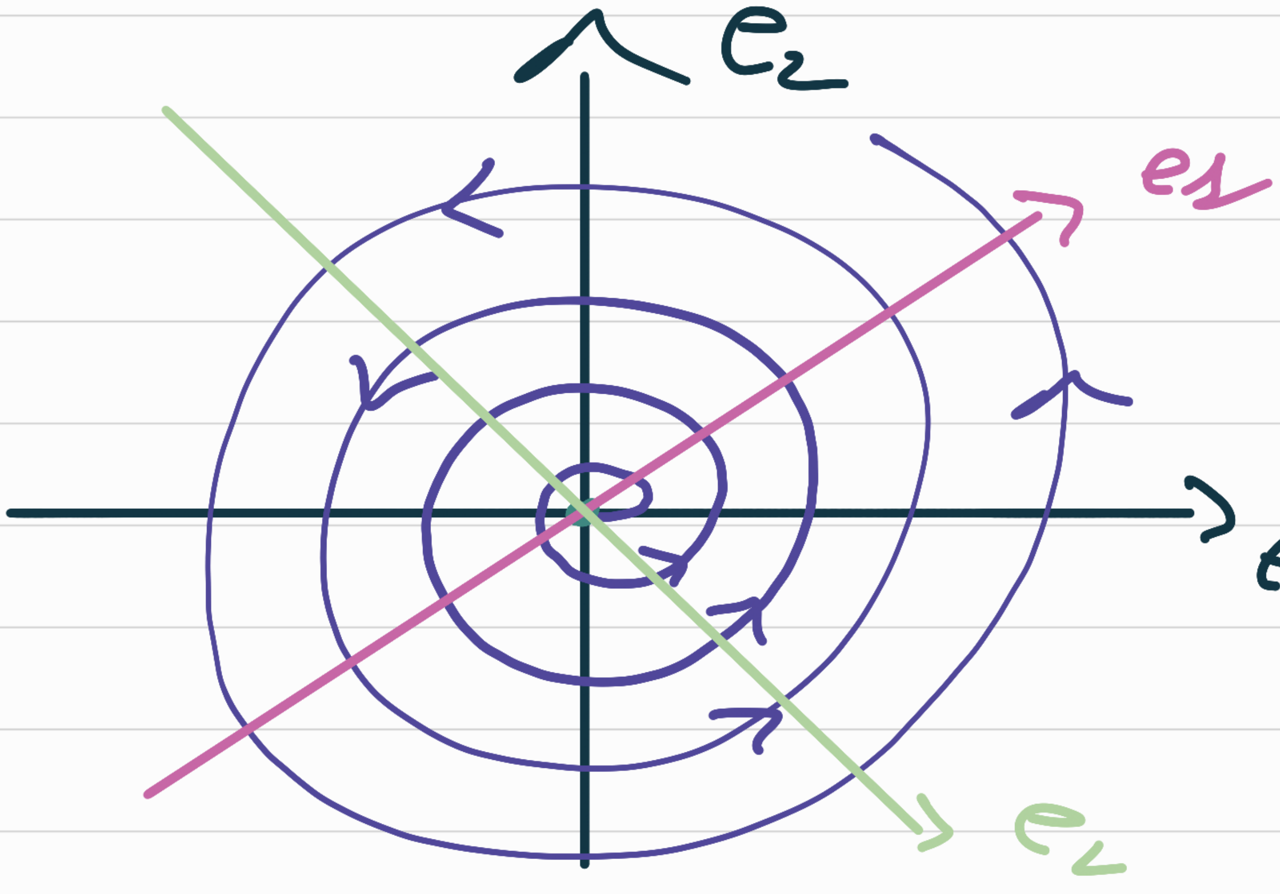


→ $\boxed{\alpha \neq 0}$

* $\alpha > 0$

spirale instable





* $\alpha < 0$