

Exercices

exercice 82: $(u^n)' = n u' u^{n-1}$

a) $f(x) = (2x - 5)^3 \rightarrow n$

$$u(x) = 2x - 5 \quad u'(x) = 2$$

$$\begin{aligned} f'(x) &= 3 \times 2 \times (2x - 5)^2 \\ &= 6(2x - 5)^2. \end{aligned}$$

b) $g(x) = (7 - 3x)^4$

$$u(x) = 7 - 3x \quad u'(x) = -3$$

$$\begin{aligned} g'(x) &= 4 \times (-3) \times (7 - 3x)^3 \\ &= -12(7 - 3x)^3 \end{aligned}$$

$$c) h(x) = (2x + 1)^{-3}$$

$$u(x) = 2x + 1 \quad u'(x) = 2$$

$$\begin{aligned} h'(x) &= (-3) \times 2 \times (2x + 1)^{-3-1} \\ &= -6 (2x + 1)^{-4} \end{aligned}$$

exercice 77:

a) • $f(x) = \frac{1}{x-2}$

$$\left(\frac{1}{u}\right)' = -\frac{u'}{u^2}$$

$$u(x) = x-2 \quad u'(x) = 1$$

$$f'(x) = -\frac{1}{(x-2)^2}$$

• $g(x) = \frac{4x-7}{x-2}$

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

$$u(x) = 4x - 7$$

$$u'(x) = 4$$

$$v(x) = x - 2$$

$$v'(x) = 1$$

$$g'(x) = \frac{4(x-2) - (4x-7) \cdot 1}{(x-2)^2}$$

$$= \frac{\cancel{4x} - 8 - \cancel{4x} + 7}{(x-2)^2}$$

$$= \frac{-1}{(x-2)^2}$$

On remarque que f et g ont la même dérivée.

$$b) f(x) - g(x)$$

$$= \frac{1}{x-2} - \frac{4x-7}{x-2}$$

$$= \frac{1 - (4x-7)}{x-2}$$

$$= \frac{1 - 4x + 7}{x-2}$$

$$= \frac{8-4x}{x-2}$$

$$u(x) = 8-4x \quad u'(x) = -4$$

$$v(x) = x-2 \quad v'(x) = 1$$

D'un côté,

$$(f(x) - g(x))'$$

$$= \frac{-4(x-2) - (8-4x)x + 1}{(x-2)^2}$$

$$= \frac{-\cancel{4}x + 8 - \cancel{8} + \cancel{4}x}{(x-2)^2}$$

$$= 0$$

De l'autre côté,

$$(f(x) - g(x))' = f'(x) - g'(x)$$

Donc on a :

$$f'(x) - g'(x) = 0$$

$$\text{et d'où } f'(x) = g'(x).$$