Worksheet 2

exercice 4 :

Spanning set $\Rightarrow$ famille génératrice

↳ Vect

1) $f(x,y,z) = (2x+y+z,\ x-y-z)$

$$\overbrace{\mathrm{Mat}_{\underset{\mathbb{R}^3}{\mathcal{B}},\underset{\mathbb{R}^2}{\mathcal{C}}}(f)}^{A} = \begin{array}{cl} \begin{pmatrix} 2 & 1 & 1 \\ 1 & -1 & -1 \end{pmatrix} & \begin{array}{l} \to e'_1 \\ \to e'_2 \end{array} \\ \begin{array}{ccc} f(e_1) & f(e_2) & f(e_3) \end{array} & \end{array}$$

$$\sim \begin{pmatrix} 2 & 1 & 0 \\ 1 & -1 & 0 \end{pmatrix}$$

$$\sim \begin{pmatrix} 2 & 3 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

donc :

$$C(A) = \mathrm{Span}\{(2;1), (3,0)\}$$

$$\mathrm{rank}(A) = \underbrace{\dim(C(A))}_{\text{nb. d'él. de Span}}$$

exercice 6 :

1) $$A = \begin{array}{cccc} \multicolumn{4}{c}{\begin{pmatrix} 1 & 1 & 2 & 3 \\ 1 & -1 & 0 & 1 \\ 1 & 1 & 2 & 3 \end{pmatrix}} \\ C_1 & C_2 & C_3 & C_4 \end{array}$$

colonnes linéai. ind.

$$\underset{3\times4}{A} = \underset{3\times2}{C}\,\underset{2\times4}{R}.$$

-> on remarque $rk(A) = 2.$

on pose $C = \begin{pmatrix} 1 & 1 \\ 1 & -1 \\ 1 & 1 \end{pmatrix}$

① $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = 1 \times \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + 0 \times \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

② $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = 0 \times \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + 1 \times \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

③ $\begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix} = 1 \times \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + 1 \times \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

④ $\begin{pmatrix} 3 \\ 1 \\ 3 \end{pmatrix} = 2 \times \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + 1 \times \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

$$R = \begin{array}{c} \begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 1 \end{pmatrix} \begin{array}{l} \rightarrow (1,1,1) \\ \rightarrow (1,-1,1) \end{array} \\ \begin{array}{cccc} c_1 & c_2 & c_3 & c_4 \end{array} \end{array}$$

exercice 7 :

1) $A = \begin{pmatrix} 1 & 1 \\ 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 1 \end{pmatrix}$

- $\mathcal{B}_{C(A)} = \{(1,1,1), (1,-1,1)\}$

dim $C(A)$ = nb. éP d'une base

- rank nullity theorem:

n.b colonne A = $\dim C(A) + \dim N(A)$

$$\dim E = \text{rank}(A) + \underbrace{\dim(\cancel{\ker}(A))}_{\text{null}(A)}$$

- $4 = 2 + \dim(N(A))$

donc $\dim(N(A)) = 4 - 2 = 2$.

- $Ax = 0$

$$\begin{cases} x + y + 2z + 3t = 0 \\ x - y + t = 0 \\ x + y + 2z + 3t = 0 \end{cases}$$

$$\iff \begin{cases} x + y + 2z + 3t = 0 \\ x - y + t = 0 \end{cases}$$

$$\iff \begin{cases} -t + y + y + 2z + 3t = 0 \\ x = -t + y \end{cases}$$

$$\iff \begin{cases} z = -y - t \\ x = -t + y \end{cases}, \quad t, y \in \mathbb{R}$$

$$(x, y, z, t) = (-t + y, y, -y - t, t)$$

$$= t(-1, 0, -1, 1)$$

$$+ y(1, 1, -1, 0)$$

donc $\mathcal{B}_{N(A)} = \{(-1, 0, -1, 1), (1, 1, -1, 0)\}$