Worksheet 4.

exercice 1 :

$$\mathcal{F} = \left\{ \begin{array}{c} u_1 \\ \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \end{array} ; \begin{array}{c} u_2 \\ \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix} \end{array} ; \begin{array}{c} u_3 \\ \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix} \end{array} \right\}$$

- orthogonal : $\langle e_i | e_j \rangle = 0 \quad i \neq j$
- orthonormal : $\langle e_i | e_j \rangle = \delta_{ij}$

$\hookrightarrow$ normaliser un vecteurs

$$e_i = \frac{u_i}{\|u_i\|}.$$

Gram - Schmidt:

- $v_1 = u_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$

- $v_2 = u_2 - \underbrace{\frac{\langle v_1 | u_2 \rangle}{\|v_1\|^2} \cdot v_1}_{P_{u_1}(u_2) \in \mathbb{R}}$

$$= \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix} - \frac{1}{2} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} -1/2 \\ 1/2 \\ -1 \\ 0 \end{pmatrix}$$

- $v_3 = u_3 - \frac{\langle v_1 | u_3 \rangle}{\|v_1\|^2} \cdot v_1$

$$- \frac{\langle v_2 | u_3 \rangle}{\|v_2\|^2} \cdot v_2$$

$$= \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix} - \frac{2}{2} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} - \frac{0}{2} \begin{pmatrix} -1/2 \\ 1/2 \\ -1 \\ 0 \end{pmatrix}$$

$$\bullet\ v_3 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\bullet\ e_1 = \frac{v_1}{\|v_1\|} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\bullet\ e_2 = \frac{v_2}{\|v_2\|} = \frac{1}{\sqrt{\frac{3}{2}}} \begin{pmatrix} -1/2 \\ 1/2 \\ -1 \\ 0 \end{pmatrix}$$

$$= \sqrt{\frac{2}{3}} \begin{pmatrix} -1/2 \\ 1/2 \\ -1 \\ 0 \end{pmatrix}$$

$$\bullet\ v_3 = \frac{v_3}{\|v_3\|} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

exercice 4 :

1) on pose $X(t) = \begin{pmatrix} y(t) \\ y'(t) \end{pmatrix}$

d'où $X'(t) = \begin{pmatrix} y'(t) \\ y''(t) \end{pmatrix}$

$$= \begin{pmatrix} y'(t) \\ -3y'(t) - 2y(t) \end{pmatrix}$$

$\begin{pmatrix} y \\ y' \end{pmatrix}$

on pose $A = \begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix}$

tel que : $X' = AX$

2) on pose $X(t) = \begin{pmatrix} y \\ y' \\ y'' \end{pmatrix}$

d'où : $X'(t) = \begin{pmatrix} y' \\ y'' \\ y''' \end{pmatrix} = \begin{pmatrix} y' \\ y'' \\ 2y''+y'-2y \end{pmatrix}$

on pose $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & 1 & 2 \end{pmatrix} \begin{pmatrix} y \\ y' \\ y'' \end{pmatrix}$

tel que $X' = AX$.

Course 1

- E finite dimension if $\dim E < +\infty$

$$\boxed{\begin{aligned} &\to \dim \mathbb{R}^n = n \\ &\to \dim \mathbb{R}_n[X] = n+1 \\ &\to \dim \mathcal{M}_{n,p}(\mathbb{R}) = n \times p. \end{aligned}}$$

- $\mathrm{Span}(\mathcal{F})$ sev de E.

- $\dim(E) = \mathrm{Card}(\mathcal{B})$, with $\mathcal{B}$ basis of E.

- $\dim E = 1 \Rightarrow$ vector line
- $\dim E = 2 \Rightarrow$ vector plan.
- F sev E et $\dim E = n$

  $\hookrightarrow \dim F \leq n$.
- $F = E \iff \begin{cases} F \text{ sev } E \\ \dim F = \dim E \end{cases}$.