

Worksheet 4.

exercice 1:

$$\mathcal{F} = \left\{ \overset{u_1}{\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}} ; \overset{u_2}{\begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix}} ; \overset{u_3}{\begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}} \right\}$$

• orthogonal : $\langle e_i | e_j \rangle = 0 \quad i \neq j$

• orthonormal : $\langle e_i | e_j \rangle = \delta_{ij}$

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normaliser un vecteurs

$$e_i = \frac{u_i}{\|u_i\|}.$$

Gram - Schmidt:

$$\cdot v_1 = u_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\cdot v_2 = u_2 - \frac{\langle v_1 | u_2 \rangle}{\|v_1\|^2} \cdot v_1$$

$\langle v_1 | u_2 \rangle \in \mathbb{R}$

$$= \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix} - \frac{1}{2} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} -1/2 \\ 1/2 \\ -1 \\ 0 \end{pmatrix}$$

$$\cdot v_3 = u_3 - \frac{\langle v_1 | u_3 \rangle}{\|v_1\|^2} \cdot v_1$$

$$- \frac{\langle v_2 | u_3 \rangle}{\|v_2\|^2} \cdot v_2$$

$$= \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix} - \frac{2}{2} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} - \frac{0}{2} \begin{pmatrix} -1/2 \\ 1/2 \\ -1 \\ 0 \end{pmatrix}$$

$$\cdot v_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\cdot e_1 = \frac{v_1}{\|v_1\|} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\cdot e_2 = \frac{v_2}{\|v_2\|} = \frac{1}{\sqrt{\frac{3}{2}}} \begin{pmatrix} -1/2 \\ 1/2 \\ -1 \\ 0 \end{pmatrix}$$

$$= \sqrt{\frac{2}{3}} \begin{pmatrix} -1/2 \\ 1/2 \\ -1 \\ 0 \end{pmatrix}$$

$$\cdot v_3 = \frac{v_3}{\|v_3\|} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

exercice 4:

1) on pose $X(t) = \begin{pmatrix} y(t) \\ y'(t) \end{pmatrix}$

d'où $X'(t) = \begin{pmatrix} y'(t) \\ y''(t) \end{pmatrix}$

$$= \begin{pmatrix} y'(t) \\ -3y'(t) - 2y(t) \end{pmatrix} \begin{pmatrix} y \\ y' \end{pmatrix}$$

on pose $A = \begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix}$

tel que : $X' = AX$

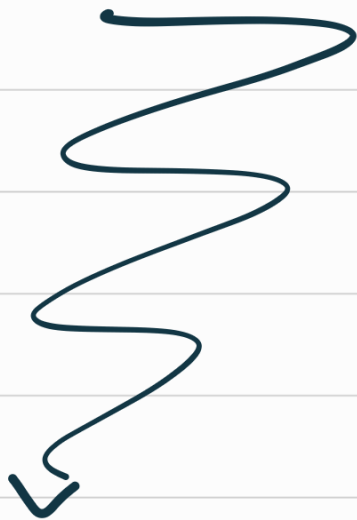


2) on pose $X(t) = \begin{pmatrix} y \\ y' \\ y'' \end{pmatrix}$

d'où: $X'(t) = \begin{pmatrix} y' \\ y'' \\ y''' \end{pmatrix} = \begin{pmatrix} y' \\ y'' \\ 2y'' + y' - 2y \end{pmatrix}$

on pose $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & 1 & 2 \end{pmatrix}$ (y, y', y'')

tel que $X' = AX$.



Course 1

- E finite dimension $\Leftrightarrow \dim E < +\infty$

$$\rightarrow \dim \mathbb{R}^n = n$$

$$\rightarrow \dim \mathbb{R}_n[X] = n+1$$

$$\rightarrow \dim_{\mathbb{R}} M_{n,p}(\mathbb{R}) = n \times p.$$

- $\text{Span}(\mathcal{B})$ sev de E .

$$\dim(E) = \text{Card}(\mathcal{B}),$$

with $\mathcal{B}$ basis of E .

• $\dim E = 1 \Rightarrow$ vector line

• $\dim E = 2 \Rightarrow$ vector plan.

• F ser E et $\dim E = n$

$\hookrightarrow \dim F \leq n$.

• $F = E \Leftrightarrow \begin{cases} F \text{ ser } E \\ \dim F = \dim E. \end{cases}$