Dérivations

• Formule Fonctions Usuelles

Fonction	Dérivée
$ax + b$	a
x^2	$2x$
x^n, $n \geqslant 2$	$n x^{n-1}$
$\sqrt{x}$	$\dfrac{1}{2\sqrt{x}}$

$\frac{1}{x}$	$-\frac{1}{x^2}$
$\frac{1}{x^n}$, $n \geq 1$	$-\frac{n}{x^{n+1}}$

- **Formules** :

→ $(u+v)' = u' + v'$

→ $(ku)' = ku'$

→ $(u \times v)' = u' \times v + u \times v'$

→ $\left(\frac{u}{v}\right)' = \frac{u' \times v - u \times v'}{v^2}$

→ $\left(\frac{1}{u}\right)' = -\frac{u'}{u^2}$

exercice :

- $f(x) = 2x^3 + 3x^2 - 12x + 1$

$\hookrightarrow f'(x) = 2 \times 3x^2 + 3 \times 2x - 12$

$= 6x^2 + 6x - 12$.

- $f(x) = -x^2 + 3x - 1$

$\hookrightarrow f'(x) = -1 \times 2x + 3$

$= -2x + 3$.

• $f(x) = 4x^3 - 5x^2 + 3x - 1$

$\hookrightarrow f'(x) = 4 \times 3x^2 - 5 \times 2x + 3$

$= 12x^2 - 10x + 3.$

• $f(x) = \frac{3 \times 1}{x^3} - \frac{1}{x}$

$\hookrightarrow f'(x) = 3 \times \frac{-3}{x^4} - \left(-\frac{1}{x^2}\right)$

$= \frac{-9}{x^4} + \frac{1}{x^2}.$

$$\bullet\ f(x) = 3x^4 + 5x - 4 + \frac{4}{x^5}$$

$$f'(x) = 3 \times 4x^3 + 5 + 4 \times \frac{-5}{x^6}$$

$$= 12x^3 + 5 - \frac{20}{x^6}.$$

$$\bullet\ f(x) = \underbrace{(2x+3)}_{u}\underbrace{(3x-7)}_{v}$$

$$u'(x) = 2 \qquad\qquad v'(x) = 3$$

$$u' \times v \quad + \quad u \times v'$$

$$f'(x) = 2 \times (3x-7) + (2x+3) \times 3$$

$$= 6x - 14 + 6x + 9$$

$$= 12x - 5.$$

$$\bullet\ f(x) = \underbrace{(x+5)}_{u}\underbrace{(x^2+1)}_{v}$$

$$f'(x) = 1 \times (x^2+1) + (x+5) \times 2x$$

$$= x^2+1 + 2x^2 + 10x$$

$$= 3x^2 + 10x + 1.$$

$$\bullet\ f(x) = \frac{-4x+1}{3x-5} \begin{matrix}]u \\ ]v \end{matrix}$$

$u'(x) = -4$ $\qquad$ $v'(x) = 3$

$$f'(x) = \frac{-4(3x-5) - (-4x+1) \times 3}{(3x-5)^2}.$$

$$f'(x) = \frac{-\cancel{12x} + 20 + \cancel{12x} - 3}{(3x-5)^2}$$

$$= \frac{17}{(3x-5)^2}.$$

- $f' > 0 \Rightarrow f$ croissante
- $f' < 0 \Rightarrow f$ décroissante

exercice :

- $f(x) = \dfrac{x^2 - 1}{2x - 1}$, $x \neq \frac{1}{2}$

1) Déterminer la dérivée

$$f'(x) = \frac{2x(2x-1) - 2(x^2-1)}{(2x-1)^2}$$

$$= \frac{4x^2 - 2x - 2x^2 + 2}{(2x-1)^2}$$

$$= \frac{2x^2 - 2x + 2}{(2x-1)^2}.$$

2) Dresser le tableau de variations de f.

$\cdot \; f'(x) = \dfrac{2x^2 - 2x + 2}{(2x-1)^2 > 0}$)) ?

$\Delta = b^2 - 4ac = (-2)^2 - 4 \times 2 \times 2$

$= 4 - 16$

$= -12 < 0$

x	$-\infty$		$1/2$		$+\infty$
f'		$+$	$\Vert$	$+$	
f	x	$\nearrow$	x $\Vert$ x	$\nearrow$	x

exercice :

$$f(x) = 2x^3 + 3x^2 - 12x + 1$$

$$f'(x) = 6x^2 + 6x - 12$$

$$\Delta = b^2 - 4ac = 6^2 - 4 \times 6 \times -12$$

$$= 324 > 0$$

$$x_1 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{-6 - 18}{2 \times 6} = -2$$

$$x_2 = \frac{-b + \sqrt{\Delta}}{2a} = \frac{-6 + 18}{2 \times 6} = 1$$

x	$-\infty$		-2		1		$+\infty$
f'		$+$	0	$-$	0	$+$	
f	×	↗	$21 \to f(-2)$	↘	$-6 \to f(1)$	↗	×