

Dérivée

Exercice 5

Déterminer l'expression des fonctions dérivées des fonctions polynomiales suivantes :

1. $f: x \mapsto -3 \cdot x + 2$

2. $g: x \mapsto 4 \cdot x^2 - 4$

3. $h: x \mapsto 2 \cdot x^2 + 3 \cdot x$

4. $j: x \mapsto 5 \cdot x^3 - 2 \cdot x^2$

5. $k: x \mapsto -2 \cdot x^2 + 2 \cdot x$

6. $\ell: x \mapsto (3 \cdot x + 11)(4 - x)$

1) $f(x) = -3x + 2$

↳ $f'(x) = -3$

2) $g(x) = 4x^2 - 4$

↳ $g'(x) = 4 \times 2x = 8x$

$$3) h(x) = 2x^2 + 3x$$

$$\begin{aligned} \hookrightarrow h'(x) &= 2 \times 2x + 3 \\ &= 4x + 3 \end{aligned}$$

$$4) j(x) = 5x^3 - 2x^2$$

$$\begin{aligned} \hookrightarrow j'(x) &= 5 \times 3x^2 - 2 \times 2x \\ &= 15x^2 - 4x \end{aligned}$$

$$5) k(x) = -2x^2 + 2x$$

$$\begin{aligned} \hookrightarrow k'(x) &= -2 \times 2x + 2 \\ &= -4x + 2. \end{aligned}$$

6) $f(x) = (3x + 11)(4 - x)$

$u = 3x + 11$ $v = 4 - x$

$u' = 3$ $v' = -1$

$$\begin{aligned} f'(x) &= u' \times v + u \times v' \\ &= 3(4 - x) + (3x + 11) \times (-1) \\ &= 12 - 3x - 3x - 11 \\ &= -6x + 1 \end{aligned}$$

Exercice 8

On souhaite déterminer les expressions des dérivées des fonctions suivantes :

$$f: x \mapsto (3 \cdot x^2 + 3 \cdot x)(2 \cdot x + 2) \quad ; \quad g: x \mapsto (2 \cdot x^2 + 1)\sqrt{x}$$

$$h: x \mapsto \frac{1}{x} \cdot (3 - x^2) \quad ; \quad j: x \mapsto \frac{2}{x} \cdot \sqrt{x}$$

$$\bullet \quad f(x) = (3x^2 + 3x)(2x + 2)$$

$u' = 3 \times 2x + 3 = 6x + 3$ $v' = 2$

$$\begin{aligned} f'(x) &= (6x + 3)(2x + 2) + (3x^2 + 3x) \times 2 \\ &= 12x^2 + 12 + 6x + 6 + 6x^2 + 6x \\ &= 18x^2 + 12x + 12. \end{aligned}$$

$$\bullet g(x) = \underbrace{(2x^2 + 1)}_u \underbrace{\sqrt{x}}_v$$

$$u' = 2 \times 2x = 4x$$

$$\rightarrow v' = \frac{1}{2\sqrt{x}}$$

$$\begin{aligned} \hookrightarrow g'(x) &= 4x \times \sqrt{x} + (2x^2 + 1) \times \frac{1}{2\sqrt{x}} \\ &= 4x\sqrt{x} + \frac{2x^2 + 1}{2\sqrt{x}} \end{aligned}$$

$$\bullet h(x) = \frac{1}{x} \underbrace{(3 - x^2)}_v$$

$$u' = -\frac{1}{x^2}$$

$$\leftarrow u$$

$$v \rightarrow v' = -2x$$

$$h'(x) = -\frac{1}{x^2} \times (3 - x^2) + \frac{1}{x} \times -2x$$

$$= \frac{-3 + x^2}{x^2} - 2$$

$$\bullet \quad z(x) = \underbrace{\frac{2}{x}}_u \cdot \underbrace{\sqrt{x}}_v$$

$$u' = 2 \times \frac{-1}{x^2} \rightarrow v' = \frac{1}{2\sqrt{x}}$$

$$= -\frac{2}{x^2}$$

$$z'(x) = -\frac{2}{x^2} \times \sqrt{x} + \frac{\cancel{2}}{x} \times \frac{1}{\cancel{2}\sqrt{x}}$$

$$= -\frac{2\sqrt{x}}{x^2} + \frac{1}{x\sqrt{x}}$$

$$1. f(x) = \frac{1}{x^3 - 4}$$

$$\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$$

$$2. g(x) = \frac{1}{1-x}$$

$$\left(\frac{1}{u}\right)' = -\frac{u'}{u^2}$$

$$3. h(x) = \frac{1}{-x^2 + 5x + 7}$$

$$1) f(x) = \frac{1}{x^3 - 4} \rightarrow u(x)$$

$$u'(x) = 3x^2$$

$$f'(x) = \frac{-3x^2}{(x^3 - 4)^2}$$

$$2) g(x) = \frac{1}{1-x}.$$

$$(3x+2)^2$$

$$\hookrightarrow 2 \times (3x+2) \times 3$$

$$\frac{1}{\sqrt{5x+1}} \rightarrow \frac{1}{2\sqrt{5x+1}} \times 5$$

$$1. f(x) = \frac{x^3}{\sqrt{x}}$$

$$2. g(x) = \frac{2\sqrt{x}}{x^2 - 4}$$

$$3. h(x) = \frac{x^4 + 1}{x^3 - 1}$$