Dérivation

Fonction	Dérivée
$ax + b$	a
x^2	$2x$
x^n	$n x^{n-1}$
$\frac{1}{x}$	$-\frac{1}{x^2}$
$\frac{1}{x^n}$	$-\frac{n}{x^{n+1}}$
$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$.

- **somme** : $(u+v)' = u' + v'$

- **multiplication par un nombre** :

$$(ku)' = ku'$$

- **produit** : $(u \times v)' = u'v + uv'$

- **inverse** : $\left(\frac{1}{u}\right)' = -\frac{u'}{u^2}$

- **quotient** : $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$

exercice :

1. $f(x) = x^2 + 1$
2. $g(x) = x + \sqrt{x}$
3. $h(x) = x^3 + x^2$
4. $i(x) = x^3 + x + \frac{1}{x^2}$
5. $j(x) = \cancel{\frac{4x+1}{x}} = 4 + \frac{1}{x}$
6. $k(x) = x^2 + x + 4 + \frac{1}{x}$

1) $f'(x) = 2x$

2) $g'(x) = 1 + \frac{1}{2\sqrt{x}}$

3) $h'(x) = 3x^2 + 2x$

4) $i'(x) = 3x^2 + 1 - \frac{2}{x^3}$

5) $j'(x) = -\frac{1}{x^2}$.

6) $k'(x) = 2x + 1 - \frac{1}{x^2}$.

exercice :

1. $f(x) = 3x^2 - 5x$

2. $g(x) = -4x^2 + 3x$

3. $h(x) = \frac{1}{2}x^2 - \frac{1}{2}x + 3$

4. $i(x) = 4x^3 - \frac{2}{3}x^2 + 6$

5. $j(x) = \dfrac{x^2 - 2x + 6}{3}$

6. $k(x) = 3x^5 + 7x^2 + 1$

1) $f'(x) = 3 \times 2x - 5 = 6x - 5.$

2) $g'(x) = -4 \times 2x + 3 = -8x + 3$

3) $h'(x) = \frac{1}{\cancel{2}} \times \cancel{2}x - \frac{1}{2} = x - \frac{1}{2}.$

4) $i'(x) = 4 \times 3x^2 - \frac{2}{3} \times 2x$

$= 12x^2 - \frac{4}{3}x$

5) $j'(x) = \frac{2x - 2}{3} = \frac{2}{3}x - \frac{2}{3}.$

6) $k'(x) = 3 \times 5x^4 + 7 \times 2x$

$= 15x^4 + 14x$

exercice :

1) $$f(x) = \overbrace{x}^{u}\underbrace{\sqrt{x}}_{v}.$$

$$u'(x) = 1 \qquad v'(x) = \frac{1}{2\sqrt{x}}.$$

$$f'(x) = \overbrace{1 \times \sqrt{x}}^{u' \times v} + \overbrace{x \times \frac{1}{2\sqrt{x}}}^{u \times v'}$$

$$= \sqrt{x} + \frac{x}{2\sqrt{x}}$$

2) $$g(x) = \overbrace{\frac{1}{x}}^{u}\underbrace{\sqrt{x}}_{v}.$$

$$u'(x) = -\frac{1}{x^2} \qquad v'(x) = \frac{1}{2\sqrt{x}}$$

$$g'(x) = -\frac{1}{x^2} \times \sqrt{x} + \frac{1}{x} \times \frac{1}{2\sqrt{x}}$$

$$= -\frac{\sqrt{x}}{x^2} + \frac{1}{2x\sqrt{x}}.$$

3) $h(x) = \overbrace{x^2}^{u}\underbrace{(2x+4)}_{v}$

$u'(x) = 2x$ $v'(x) = 2$

$$h'(x) = 2x(2x+4) + 2x^2$$

$$= 4x^2 + 8x + 2x^2$$

$$= 6x^2 + 8x.$$

exercice :

$$\left(\frac{1}{u}\right)' = -\frac{u'}{u^2}$$

1. $f(x) = \dfrac{1}{x^3 - 4}$

u

$u'(x) = 3x^2$

$$f'(x) = \frac{-3x^2}{(x^3-4)^2}.$$

2. $g(x) = \dfrac{1}{1 - x}$ $\rightarrow u' = -1$

$$g'(x) = \frac{-(-1)}{(1-x)^2} = \frac{1}{(1-x)^2}$$

$$3.\ h(x) = \frac{1}{\underbrace{-x^2+5x+7}}$$

$u'(x) = -2x+5.$

$$h'(x) = \frac{-(-2x+5)}{(-x^2+5x+7)^2} = \frac{2x-5}{(-x^2+5x+7)^2}$$

exercice :

$$1.\ f(x) = \frac{x^3}{\sqrt{x}}$$ u v

$u'(x) = 3x^2$

$v'(x) = \frac{1}{2\sqrt{x}}$.

$$f'(x) = \frac{3x^2\sqrt{x} - \frac{1}{2\sqrt{x}} \times x^3}{\cancel{(\sqrt{x})^2}}$$

$$= \frac{3x^2\sqrt{x} - \frac{x^3}{2\sqrt{x}}}{x}$$

$$2.\ g(x) = \frac{2\sqrt{x}}{x^2 - 4}$$ u v

$$u'(x) = \cancel{2} \times \frac{1}{\cancel{2}\sqrt{x}} = \frac{1}{\sqrt{x}}$$

$$v'(x) = 2x$$

$$g'(x) = \frac{\frac{1}{\sqrt{x}} \times (x^2 - 4) - 2x \times 2\sqrt{x}}{(x^2 - 4)^2}$$

$$= \frac{\frac{x^2 - 4}{\sqrt{x}} - 4x\sqrt{x}}{(x^2 - 4)^2}.$$

3. $h(x) = \dfrac{x^4 + 1}{x^3 - 1}$

$u'(x) = 4x^3$

$v'(x) = 3x^2$

$$h'(x) = \frac{4x^3(x^3 - 1) - 3x^2(x^4 + 1)}{(x^3 - 1)^2}$$

$$= \frac{4x^6 - 4x^3 - 3x^6 - 3x^2}{(x^3 - 1)^2}$$

$$= \frac{x^6 - 4x^3 - 3x^2}{(x^3 + 1)^2}.$$

exercice :

- $f(x) = \overbrace{(3x^2 + 3x)}^{u}\underbrace{(2x + 2)}_{v}$

$u'(x) = 3 \times 2x + 3 = 6x + 3$

$v'(x) = 2$

$$f'(x) = (6x+3)(2x+2) + 2(3x^2+3x)$$

$$= 12x^2 + 12x + 6x + 6 + 6x^2 + 6x$$

$$= 18x^2 + 24x + 6.$$

$$\bullet \; g(x) = \frac{10x^2 + 1}{\sqrt{x}} .$$

$$u'(x) = 10 \times 2x = 20x$$

$$v'(x) = \frac{1}{2\sqrt{x}} .$$

$$g'(x) = \frac{20x\sqrt{x} - \frac{1}{2\sqrt{x}} \times (10x^2+1)}{(\cancel{\sqrt{}}x)^{\cancel{2}}}$$

$$= \frac{20x\sqrt{x} - \frac{10x^2+1}{2\sqrt{x}}}{x} .$$