

Logarithme

exercice 18:

$$f(x) = x - P_n(x), \quad x > 0$$

$$1) \quad P_n'(x) = \frac{1}{x}$$

$$P_n(u) = \frac{u'}{u}$$

$$\begin{aligned} f'(x) &= 1 - \frac{1}{x} \\ &= \frac{x - 1}{x} \end{aligned}$$

$$2) f'(x) = \frac{x-1}{x} > 0 ?$$

$$x-1=0 \Leftrightarrow x=1$$

x	0	1	$+\infty$
f'		-	+
f		$+\infty$	$+\infty$

Diagram showing the behavior of the function f near the critical point $x=1$. The function decreases from $+\infty$ at $x=0$ to a minimum at $x=1$, and then increases towards $+\infty$ as $x \rightarrow +\infty$.

$$\lim_{x \rightarrow 0^+} f(x) = -\infty$$

$$\lim_{x \rightarrow +\infty} f(x) = +\infty$$

$$f(x) = x \left(1 - \frac{\ln(x)}{x} \right)$$

or $\lim_{x \rightarrow +\infty} \frac{\ln(x)}{x} = 0$ par croissances comparées

d'où $\lim_{x \rightarrow +\infty} 1 - \frac{\ln(x)}{x} = 1$

et $\lim_{x \rightarrow +\infty} x = +\infty$.

Donc par produit, $\lim_{x \rightarrow +\infty} f(x) = +\infty$

3) $f''(x) = \frac{x - (x-1)}{x^2}$

$= \frac{1}{x^2} > 0$
donc f convexe.

exercice 24:

1) a) $\mathcal{B}(x) = x + 4e^{-x} - 5.$

$$\begin{aligned}\mathcal{B}'(x) &= 1 + 4x - e^{-x} \\ &= 1 - 4e^{-x}\end{aligned}$$

b) $\mathcal{B}'(x) \Rightarrow 0$

$$1 - 4e^{-x} \Rightarrow 0$$

$$-4e^{-x} \Rightarrow -1$$

$$e^{-x} \leq \frac{1}{4}$$

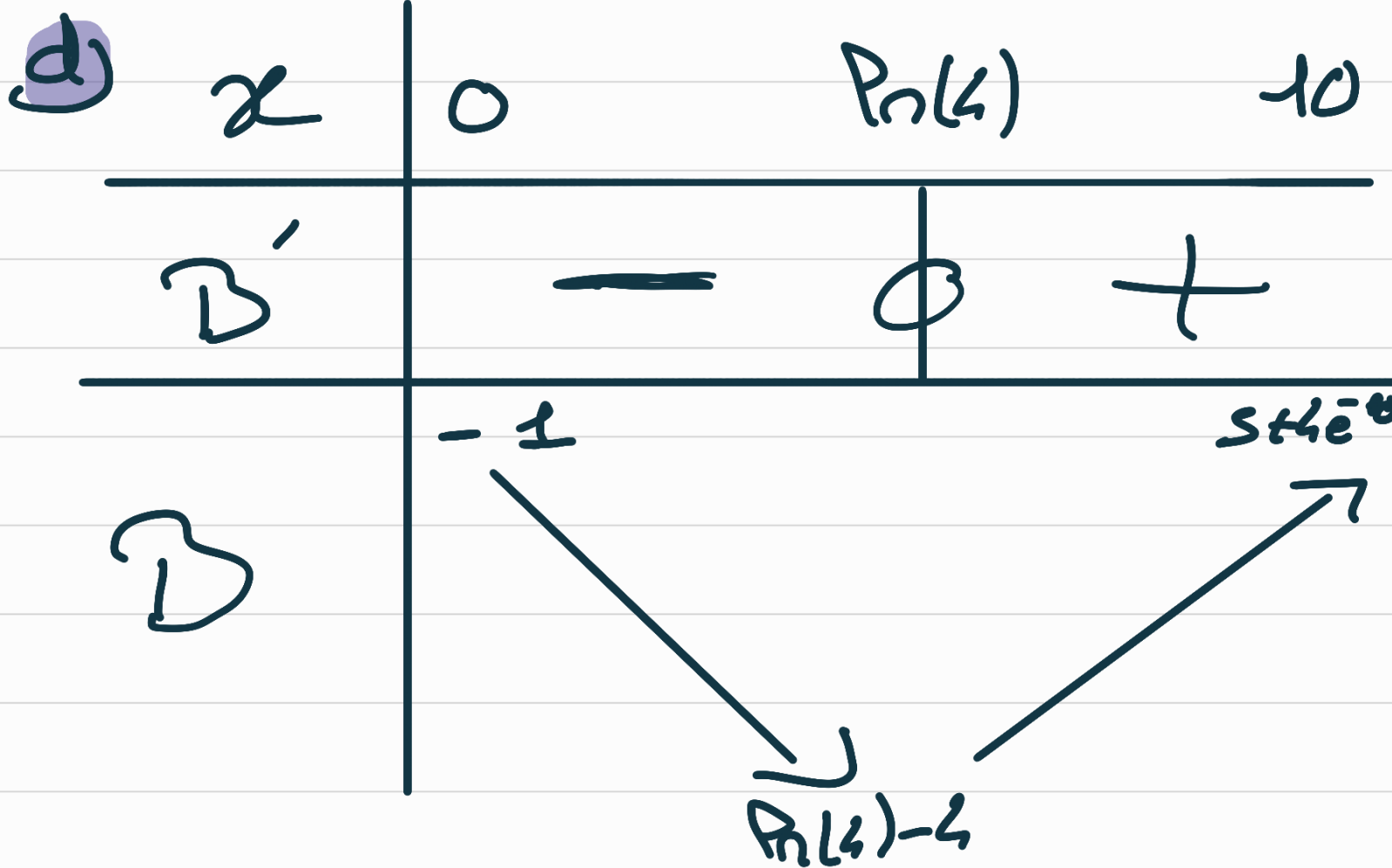
$$-x \leq \ln\left(\frac{1}{4}\right)$$

$$x \Rightarrow -\ln\left(\frac{1}{4}\right) \Rightarrow \ln(4).$$

$$\begin{aligned}
 c) \cdot D(0) &= 0 + 4e^0 - 5 \\
 &= 4 \times 1 - 5 \\
 &= -1
 \end{aligned}$$

$$\begin{aligned}
 \cdot D(10) &= 10 + 4e^{-10} - 5 \\
 &= 5 + 4e^{-10}
 \end{aligned}$$

$$\begin{aligned}
 \cdot D(h(4)) &= h(4) + 4e^{-h(4)} - 5 \\
 &= h(4) + 4e^{-h(\frac{1}{4})} - 5 \\
 &= h(4) + 4 \times \frac{1}{4} - 5 \\
 &= h(4) - 4
 \end{aligned}$$



exercice 20:

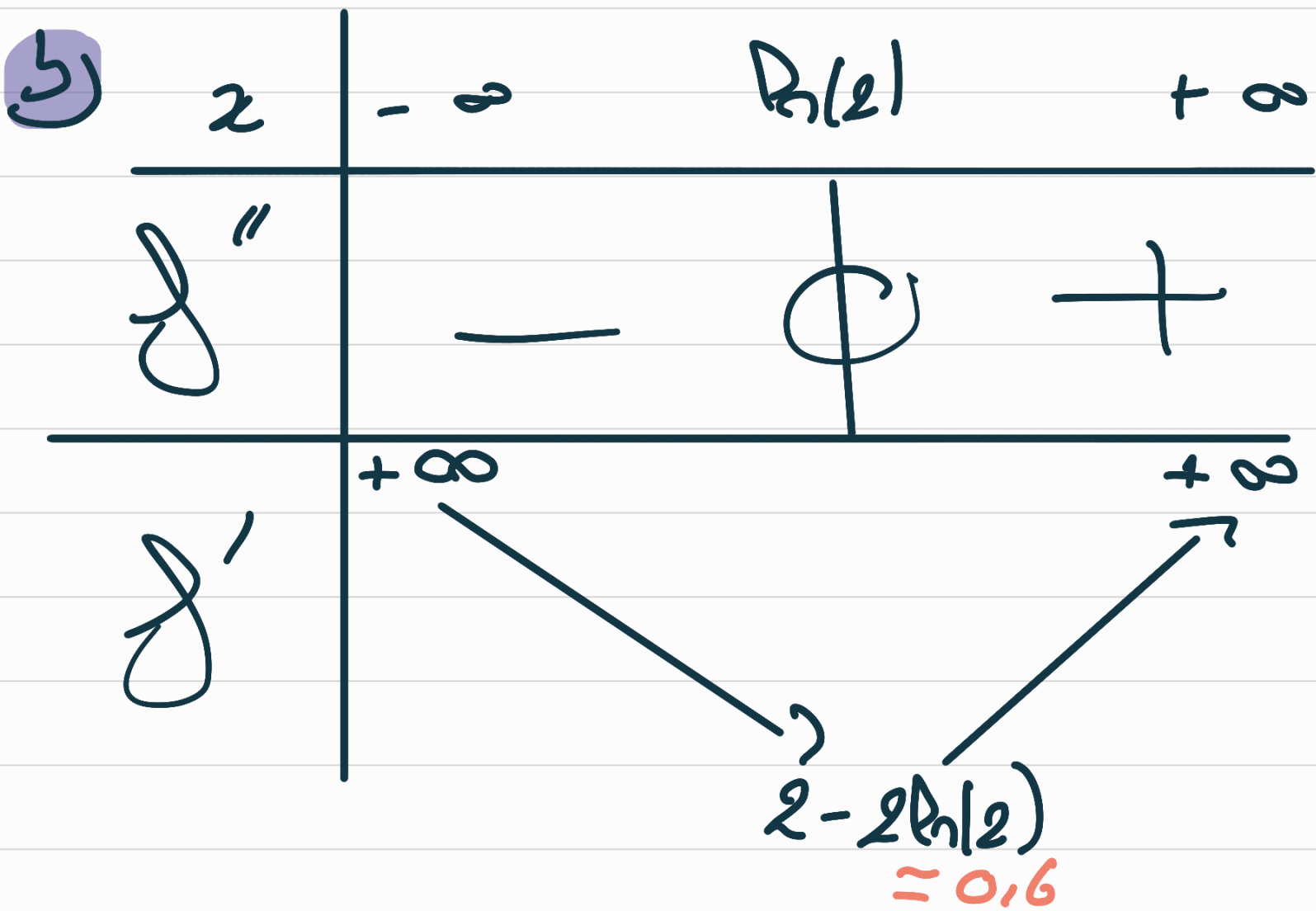
2) a) $f''(x) = e^x - 2$

$$e^x - 2 > 0$$

$$e^x > 2$$

$$x > \ln(2).$$

x	$-\infty$	$\ln(2)$	$+\infty$
f''	$=$	$\circ$	$+$



$$f'(x) = e^x - 2x$$

$$f'(\ln(2)) = e^{\ln(2)} - 2\ln(2)$$

$$= 2 - 2\ln(2)$$

$$e^x \left(1 - \frac{2x}{e^x} \right)$$

$+\infty$ $\underbrace{\quad}_{+1}$

or $\lim_{x \rightarrow +\infty} \frac{2x}{e^x} = 0$ par croissance comparée

donc $\lim_{x \rightarrow +\infty} f'(x) = +\infty$

Donc f' est positive sur $\mathbb{R}$

