# Les vecteurs

- $\vec{AB} \cdot \vec{AC} = AB \times AC \times \cos(\widehat{BAC})$

- $\vec{AB} \cdot \vec{AC} = x_{AB} x_{AC} + y_{AB} y_{AC}$

- $\vec{AB} \cdot \vec{AC} = \frac{1}{2}\left(AB^2 + AC^2 - BC^2\right)$

$$\vec{u} \cdot \vec{v} = \frac{1}{2}\left(\|\vec{u}\|^2 + \|v\|^2 - \|u-v\|^2\right)$$

$$= \frac{1}{2}\left(\|u+v\|^2 - \|u\|^2 - \|v\|^2\right)$$

$$= \frac{1}{4}\left(\|u+v\|^2 - \|u-v\|^2\right)$$

## exercice :

$A(3;2)$ et $B(-5;-3)$

$C(-1;-0,5)$

Calculer $\vec{AB}.\vec{AC}$.

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$\vec{AB}(-8;-5)$

$\vec{AC}(-4;-2,5)$

$\vec{AB}.\vec{AC} = -8\times-4 + -5\times-2,5$

$= 32 + 12,5$

$= 44,5$.

# exercice :

Soit ABC un triangle tel que :

$AB = 5$ ; $AC = 3$ ; $\widehat{BAC} = \frac{3\pi}{4}$

Calculer $\vec{AB} \cdot \vec{AC}$

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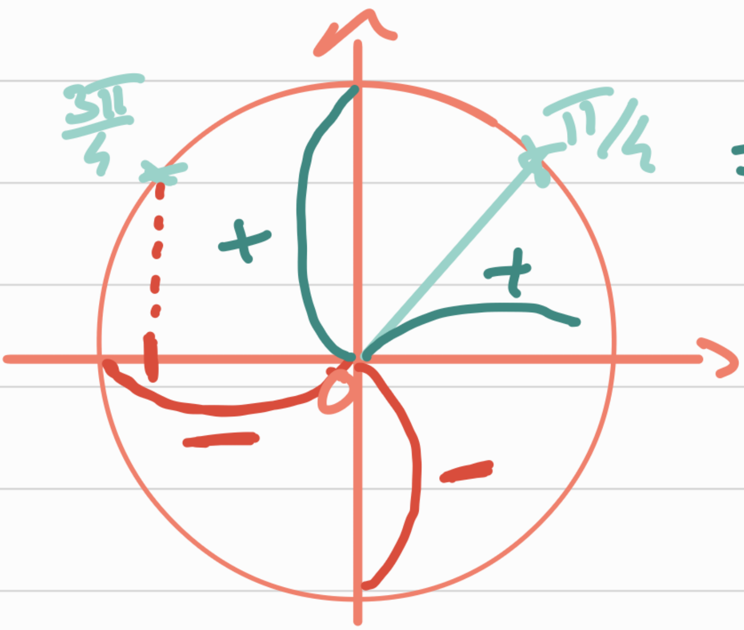
$$\vec{AB} \cdot \vec{AC} = AB \times AC \times \cos(\widehat{BAC})$$

$$= 5 \times 3 \times \cos\left(\frac{3\pi}{4}\right)$$




$$= 15 \times -\frac{\sqrt{2}}{2}$$

$$= -\frac{15\sqrt{2}}{2}.$$

# exercice :

$\|\vec{u}\| = 2 \qquad \|\vec{v}\| = 4$

$\|\vec{u} + \vec{v}\| = 7$

Calculer $\vec{u} \cdot \vec{v}$.

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$$\vec{u} \cdot \vec{v} = \frac{1}{2}\left(\|\vec{u}\|^2 + \|\vec{v}\|^2 - \|\vec{u}+\vec{v}\|^2\right)$$

$$= \frac{1}{2}(4 + 16 - 49)$$

$$= \frac{-29}{2}$$

$$= -14,5.$$

exercice :

$\|\vec{u}\| = 2$ , $\vec{u}.\vec{v} = -7$

et $(\vec{u}, \vec{v}) = \frac{\pi}{6}$ .

Détermine $\|v\|$ .

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$\vec{u}.\vec{v} = \|\vec{u}\| \times \|\vec{v}\| \times \cos(\vec{u}, \vec{v})$

$-7 = 2 \times \|\vec{v}\| \times \cos\left(\frac{\pi}{6}\right)$

$-7 = \cancel{2} \times \|\vec{v}\| \times \frac{\sqrt{3}}{\cancel{2}}$

$-\frac{7}{\sqrt{3}} = \|\vec{v}\|$

# exercice :

$M(1;3)$ $N(4;-2)$

$P(2;-1)$.

1) Déterminer $\vec{MN} \cdot \vec{MP}$

$\vec{MN}(3;-5)$ $\vec{MP}(1;-4)$

$\vec{MN} \cdot \vec{MP} = 3 \times 1 + -5 \times -4 = 23$

2) En déduire une valeur approchée de $\widehat{NMP}$ au degré.

$\vec{MN} \cdot \vec{MP} = MN \times MP \times \cos(\widehat{NMP})$

$MN = \sqrt{3^2 + (-5)^2} = \sqrt{34}$

$MP = \sqrt{1^2 + (-4)^2} = \sqrt{17}$.

$$23 = \sqrt{34} \times \sqrt{17} \times \cos(\widehat{NMP})$$

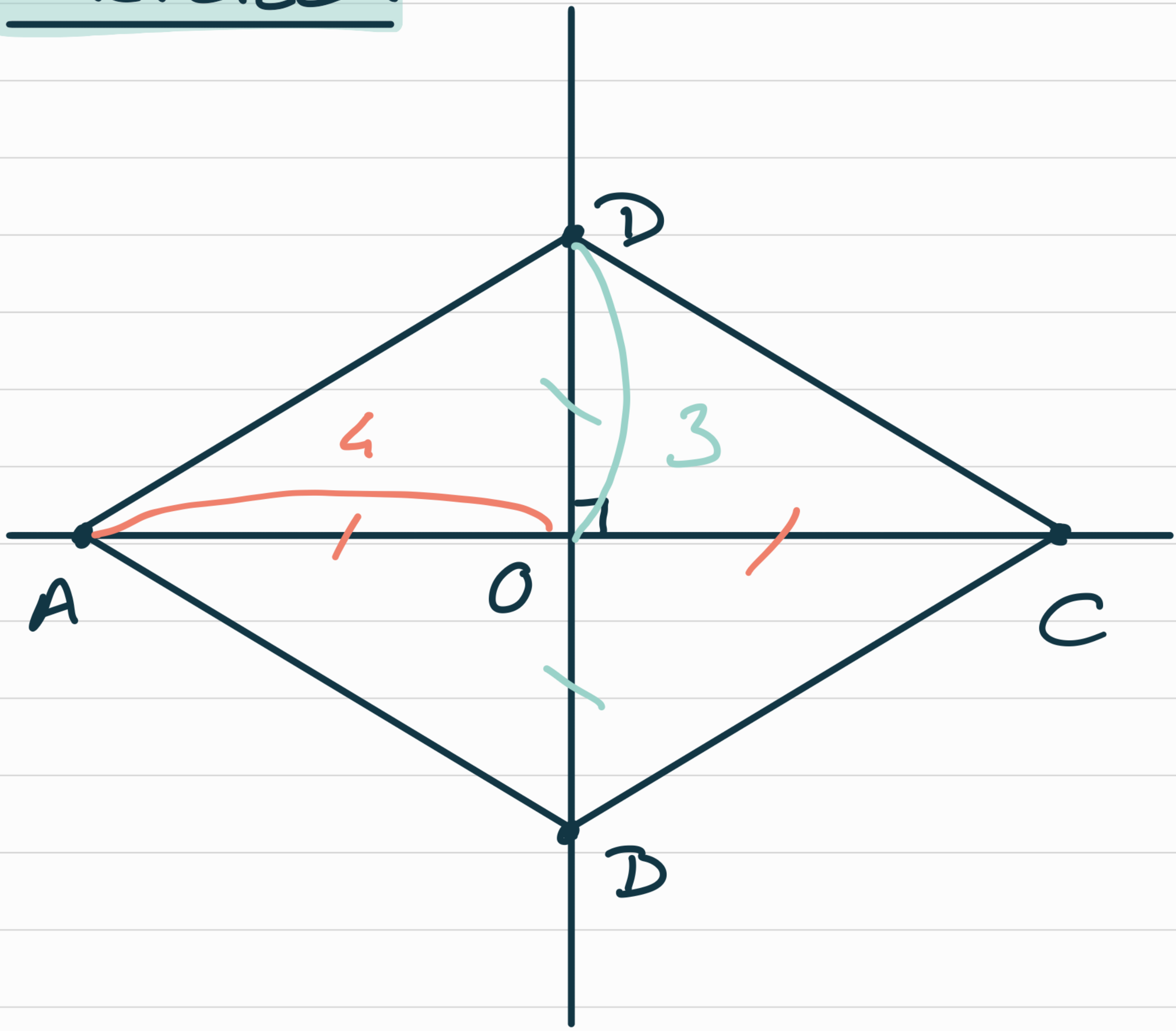
$$\frac{23}{\sqrt{34} \times \sqrt{17}} = \cos(\widehat{NMP})$$

$$\frac{23}{17\sqrt{2}} = \cos(\widehat{NMP})$$

$\rightarrow \frac{1}{2} \; ; \; -\frac{\sqrt{3}}{2} \; ; \; \frac{\sqrt{2}}{2}$

$$\Rightarrow \widehat{NMP} \simeq 17°.$$

## exercice :




- $\overrightarrow{AC} \cdot \overrightarrow{AD} = \overrightarrow{AC} \cdot \overrightarrow{AO}$

$= AC \times AO \times 1$

$= 8 \times 4 \times 1$

$= 32$


$$\overrightarrow{AB} \cdot \overrightarrow{AC} = \overrightarrow{AH} \cdot \overrightarrow{AC}$$

$$= AH \times AC$$

- $\overrightarrow{BO} \cdot \overrightarrow{BC} = \overrightarrow{BO} \cdot \overrightarrow{BO}$

$$= BO \times BO \times 1$$

$$= 3 \times 3 \times 1$$

$$= 9.$$

- $\vec{BC} \cdot \vec{BD}$

$= \vec{BO} \cdot \vec{BD}$

$= BO \times BD$

$= 3 \times 6$

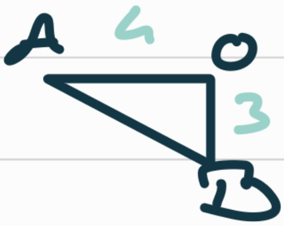
$= 18.$

- $\vec{AB} \cdot \vec{DC}$

$= \vec{AD} \cdot \vec{AB}$

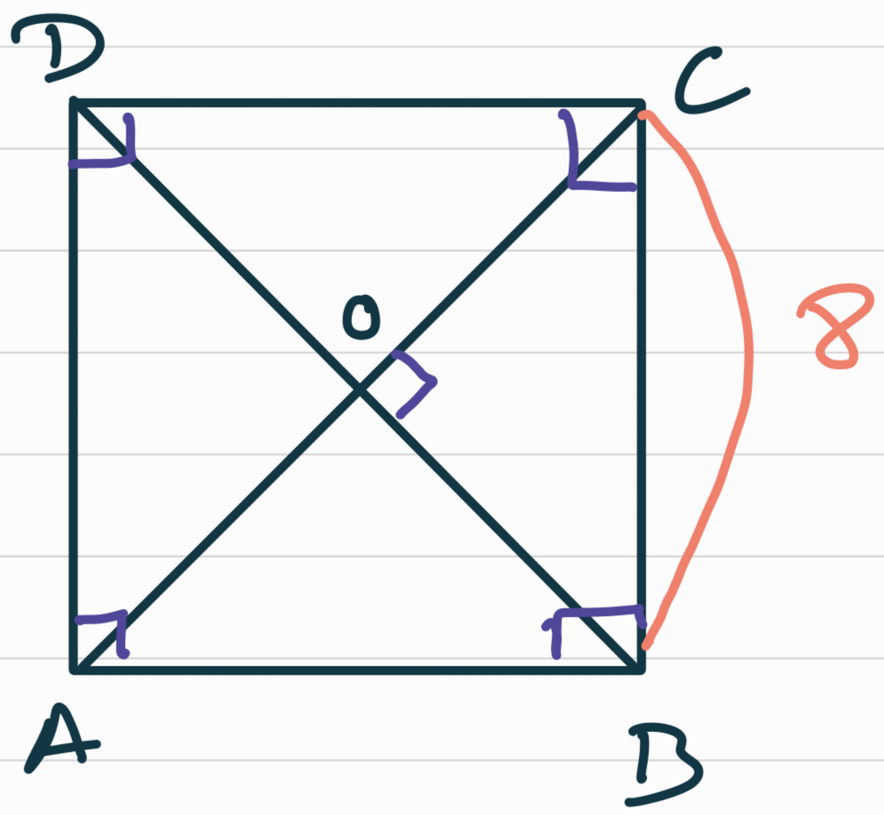
$= AB \times AB$

$= 5 \times 5 \longrightarrow$ Pythagore




$= 25$

## exercice :




• $\overrightarrow{AB} \cdot \overrightarrow{AD} = \overrightarrow{AB} \cdot \underbrace{\overrightarrow{AA}}_{=0}$

$= 0$