

Valeur absolue

$$\bullet |x| = \begin{cases} x & , x \geq 0 \\ -x & , x < 0 \end{cases}$$

exemple: $|2| = 2$

$$|-2| = 2$$

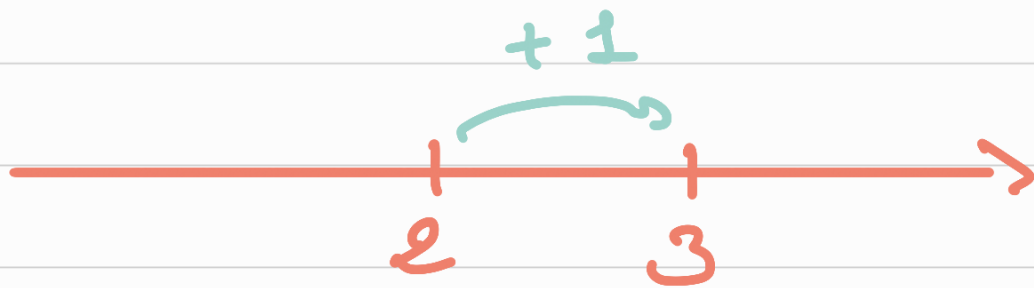
$$|-1,5| = 1,5$$

$\Rightarrow$ La valeur absolue est une distance sur $\mathbb{R}$.

$$|x| \geq 0$$

exemple : $|3 - 2| = |1| = 1$

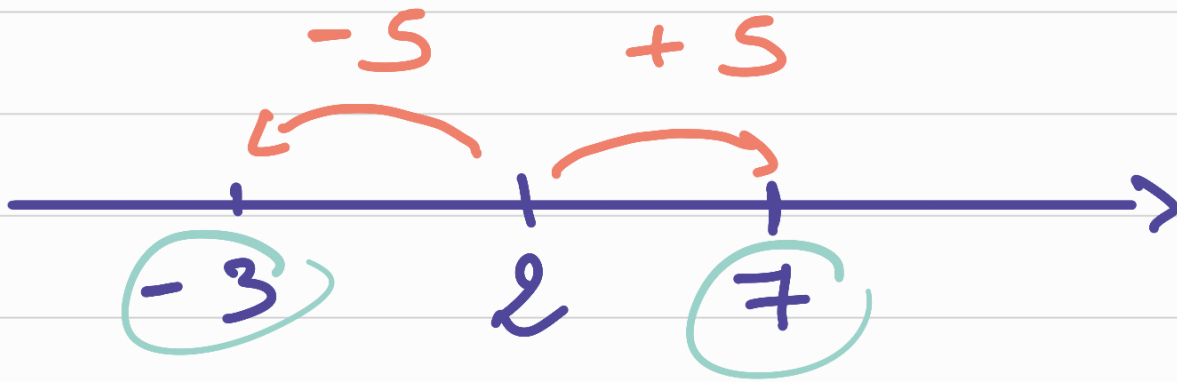
La distance entre 3 et 2 vaut 1



def: La distance entre un point d'abscisse x et un point d'abscisse a est $|x - a|$.

méthode : résoudre une équation avec la valeur absolue.

• $|x - 2| = 5$.



done $\mathcal{S} = \{-3; 7\}$.

$$|-3-2| = |-5| \\ = 5 \checkmark$$

$$|7-2| = |5| \\ = 5 \checkmark$$



$$|x+2| = |x-(-2)|$$

Exercices

exercice:

Donner la valeur absolue:

- $|1-2| = 2$

- $|3| = 3$

- $|\frac{2}{3}| = \frac{2}{3}$

- $|\sqrt{2}| = \sqrt{2}$

- $|\frac{-8}{7}| = \frac{8}{7}$

- $|3\pi| = 3\pi$.

exercice:

Donner la valeur absolue de la distance entre deux points

• $A(3)$ et $B(5)$

$$AB = |3 - 5| = |-2| = 2.$$

• $A(-4)$ et $B(5)$

$$AB = |-4 - 5| = |-9| = 9$$

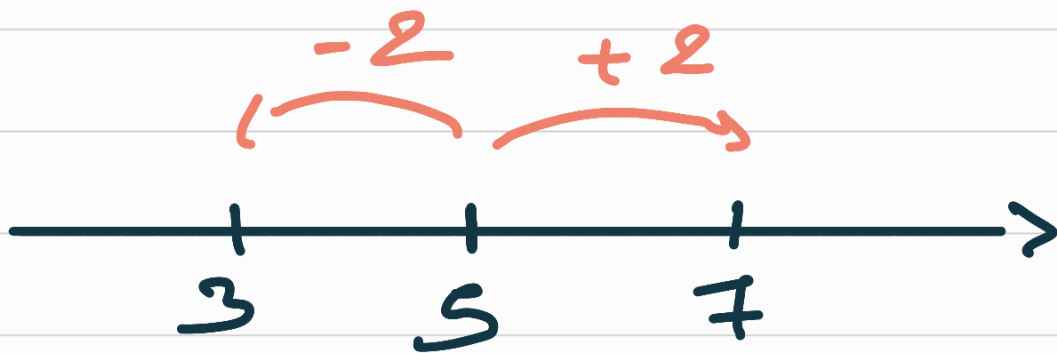
• $A(-2)$ et $B(-7)$

$$AB = |-7 - (-2)| = |-5| = 5.$$

exercice :

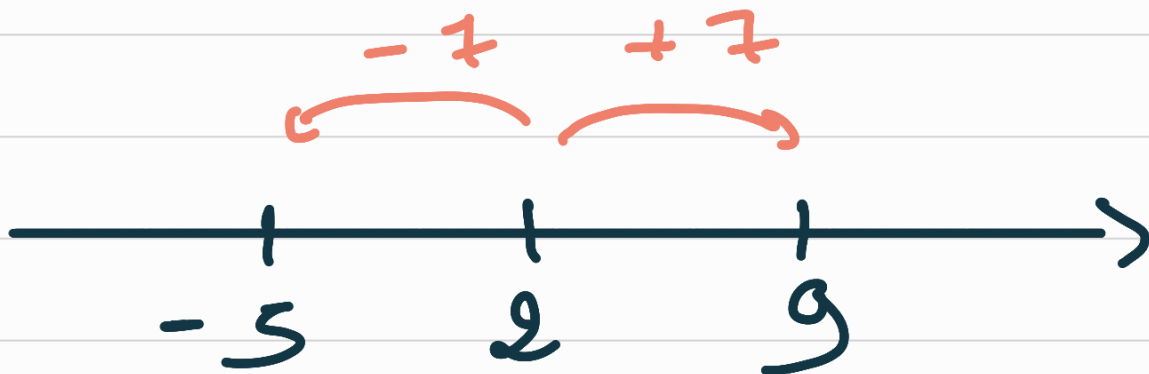
Résoudre les équations

• $|x - 5| = 2$



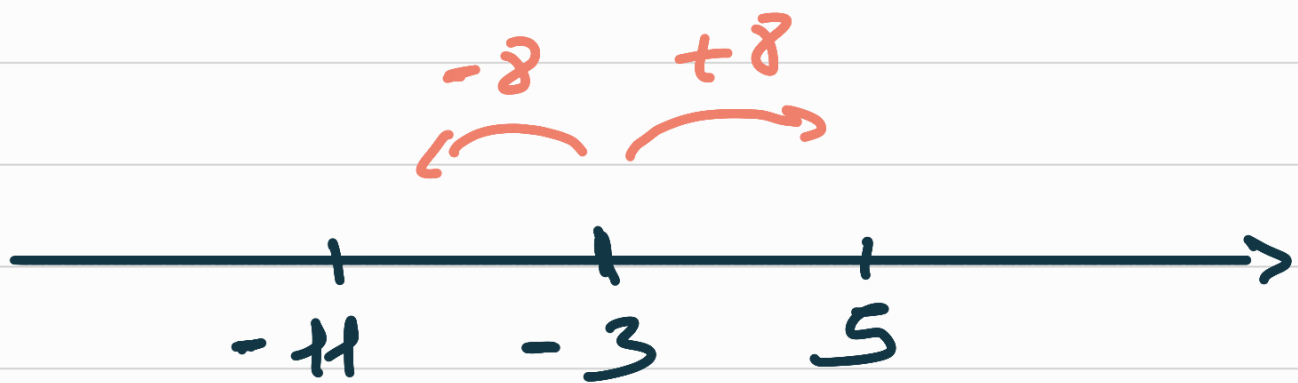
donc $S = \{3; 7\}$.

• $|x - 2| = 7$

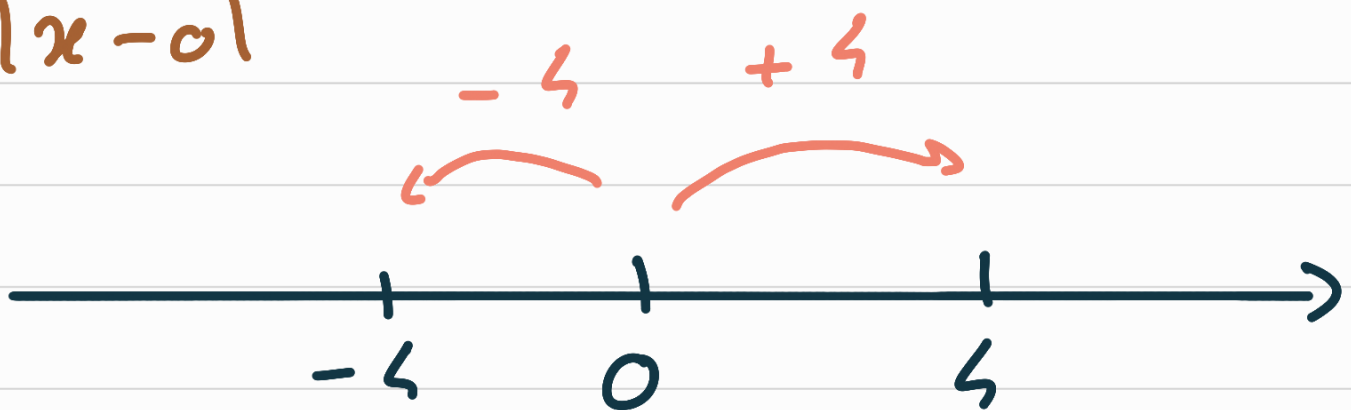


donc $S = \{-5; 9\}$.

$$\bullet |x + 3| = 8$$



$$\bullet \underbrace{|x|}_{|x-0|} = 4$$



done $\mathcal{S} = \{-4; 4\}.$

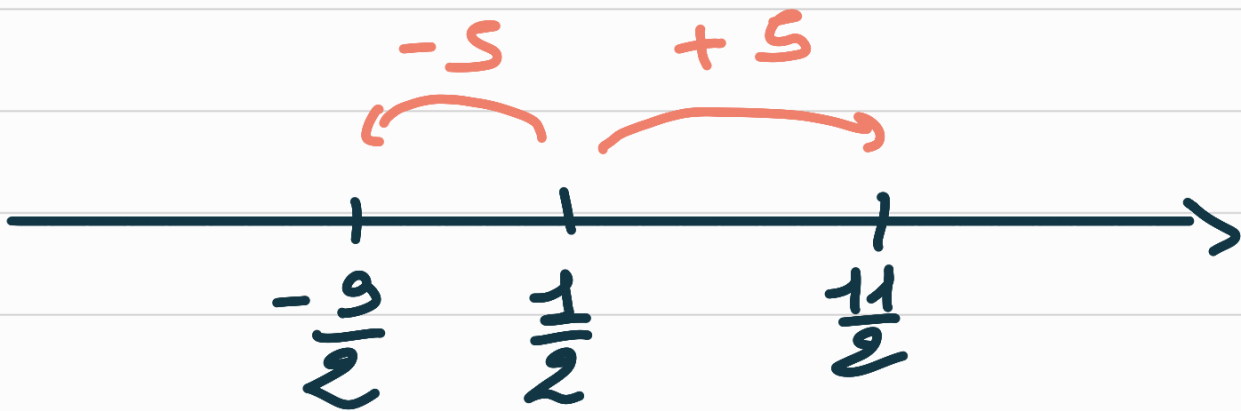
$$\bullet |2x - 4| = -1$$

$$\text{done } \mathcal{S} = \emptyset.$$

$$\bullet \left| \frac{1}{2} - x \right| = 5$$



$$\left| -\left(x - \frac{1}{2}\right) \right| = \left| x - \frac{1}{2} \right|$$



$$\text{done } \mathcal{S} = \left\{ -\frac{9}{2}; \frac{11}{2} \right\}.$$

$$\bullet |5x + 7| = 10$$

$$5x + 7 = 10$$

$$5x = 10 - 7$$

$$5x = 3$$

$$x = \frac{3}{5}$$

$$5x + 7 = -10$$

$$5x = -10 - 7$$

$$5x = -17$$

$$x = -\frac{17}{5}$$

$$\text{hence } \mathcal{S} = \left\{ \frac{3}{5}; -\frac{17}{5} \right\}.$$

$$\circ |4x - 1| = |2x - 3|$$

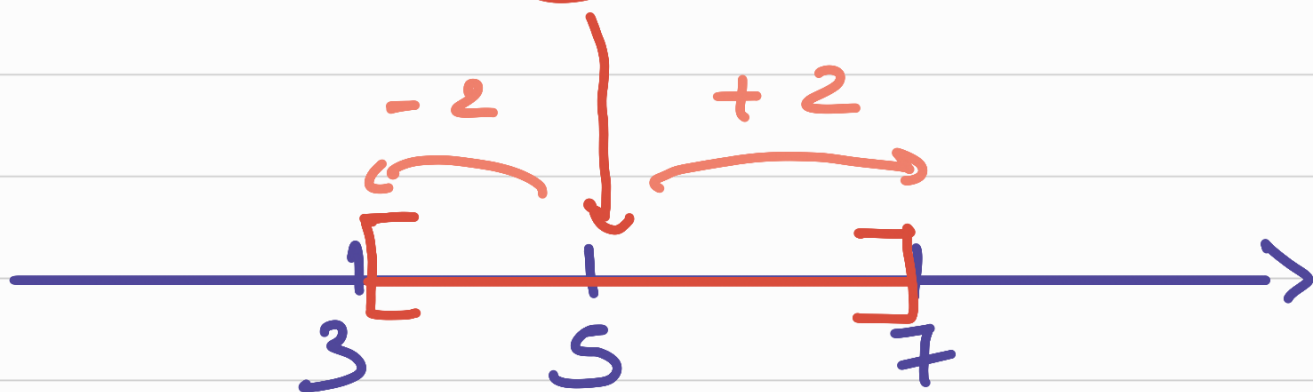
$$\left. \begin{array}{l} 4x - 1 = 2x - 3 \\ 4x - 2x = -3 + 1 \\ 2x = -2 \\ x = -1 \end{array} \right\} \begin{array}{l} 4x - 1 = -(2x - 3) \\ 4x - 1 = -2x + 3 \\ 4x + 2x = 3 + 1 \\ 6x = 4 \\ x = \frac{4}{6} = \frac{2}{3} \end{array}$$

$$\text{done } \Rightarrow = \left\{ -1; \frac{2}{3} \right\}.$$

$$|A| = |B| \Leftrightarrow A = B \text{ or } A = -B$$

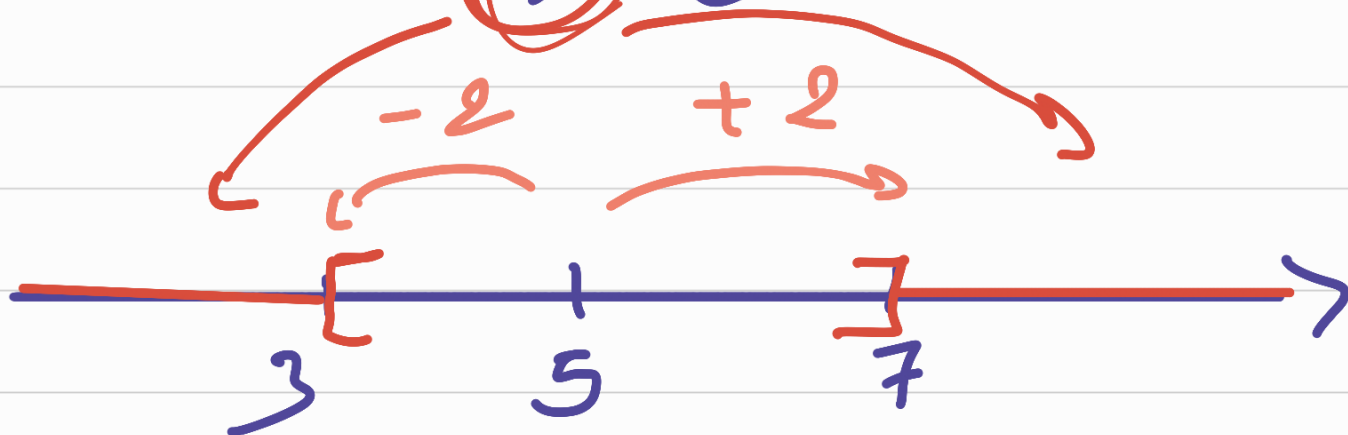
méthode : résoudre des inéquations
avec la valeur absolue.

• $|x - 5| \leq 2$



donc $\mathcal{S} = [3; 7]$

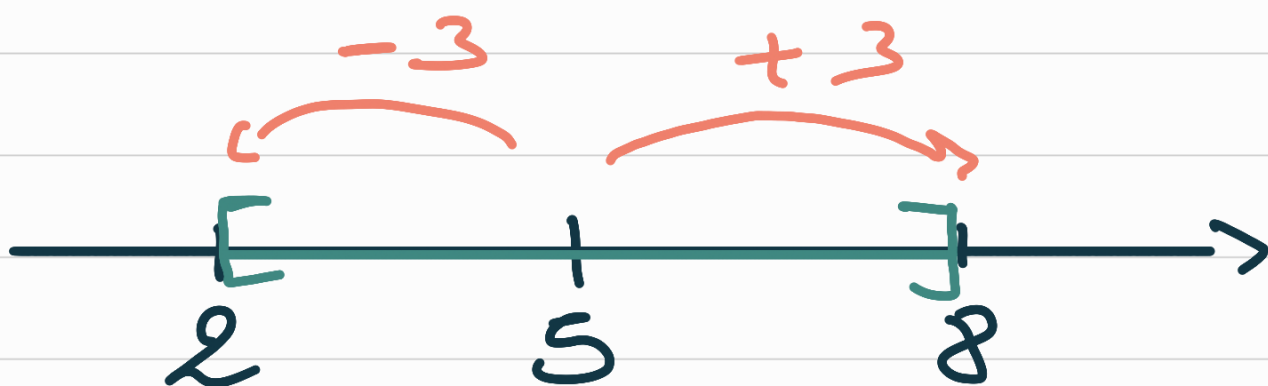
• $|x - 5| > 2$



done $\mathcal{S} =]-\infty; 3[\cup]7; +\infty[$

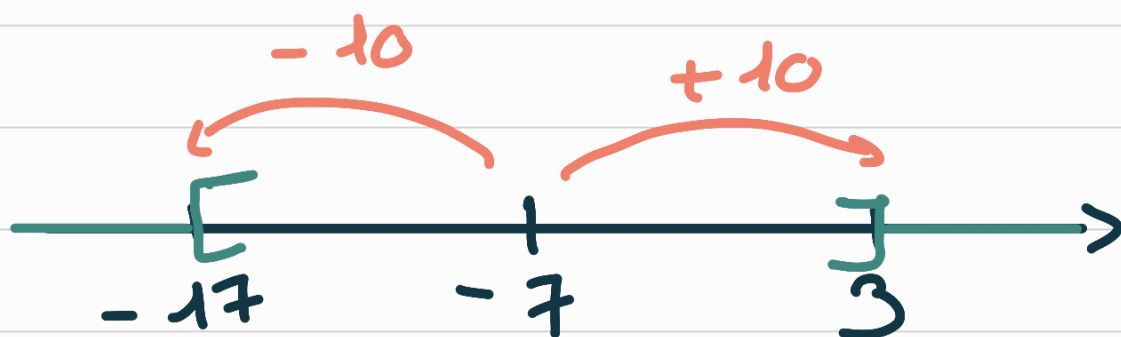
example:

• $|x - 5| \leq 3$



done $\mathcal{S} = [2; 8]$.

• $|x + 7| > 10$



done $\mathcal{D} =]-\infty; -17[\cup]3; +\infty[.$