Second degré

exercice :

1) Donne la forme canonique de :

$$f(x) = 5x^2 - 3x - 2.$$

$$f(x) = 5\left(\underbrace{x^2 - \frac{3}{5}x}_{a^2 - 2ab} - \frac{2}{5}\right)$$

$$= 5\left(x^2 - 2\times\frac{3}{5\times2}x - \frac{2}{5}\right)$$

$$= 5\left(\overbrace{x^2 - 2\times\frac{3}{10}x}^{\left(x - \frac{3}{10}\right)^2} - \frac{2}{5}\right)$$

$$= 5\left(\left(x-\frac{3}{10}\right)^2 - \left(\frac{3}{10}\right)^2 - \frac{2}{5}\right)$$

$$= 5\left(\left(x-\frac{3}{10}\right)^2 - \frac{9}{100} - \frac{2}{5}\right)$$

$$= 5\left(\left(x-\frac{3}{10}\right)^2 - \frac{49}{100}\right)$$

$$5 \times \frac{49}{100}$$

$$= 5\left(x-\frac{3}{10}\right)^2 - \frac{49}{20}$$

$$\alpha = -\frac{b}{2a} = -\frac{(-3)}{2\times 5} = \frac{3}{10}$$

$$\beta = f(\alpha) = 5\times\left(\frac{3}{10}\right)^2 - 3\times\frac{3}{10} - 2$$

$$= 5\times\frac{9}{100} - \frac{9}{10} - 2$$

$$= \frac{9}{20} - \frac{9}{10} - 2$$

$$= \frac{9}{20} - \frac{18}{20} - \frac{40}{20}$$

$$= -\frac{49}{20}$$

2) En déduire le tableau de variations.

x	$-\infty$		$\frac{3}{10}$		$+\infty$
f		$\searrow$		$\nearrow$	
			$-\frac{49}{20}$		

$a > 0$

exercice :

Résoudre l'équation :

- $8x^2 - 4x + 2 = \frac{3}{2}$

$$8x^2 - 4x + 2 - \frac{3}{2} = 0$$

$$f(x) = 8x^2 - 4x + \frac{1}{2} = 0$$

$$\Delta = b^2 - 4ac = (-4)^2 - 4 \times 8 \times \frac{1}{2}$$

$$= 16 - 16 = 0$$

$$x_0 = -\frac{b}{2a} = \frac{4}{8 \times 2} = \frac{1}{4}.$$

donc $S = \{\frac{1}{4}\}$. $\longrightarrow f(x) = 8(x - \frac{1}{4})^2$

$a(x - x_0)^2$

$$\bullet\ \underbrace{2x^2 - 8x + 6}_{f(x)} = 0$$

$$\Delta = b^2 - 4ac = (-8)^2 - 4\times 2\times 6$$

$$= 64 - 48$$

$$= 16 > 0$$

$$x_1 = \frac{-b-\sqrt{\Delta}}{2a} = \frac{8-4}{4} = 1$$

$$x_2 = \frac{-b+\sqrt{\Delta}}{2a} = \frac{8+4}{4} = 3.$$

donc $S = \{1 ; 3\}$.

$\hookrightarrow f(x) = 2(x-1)(x-3).$

exercice :

Résoudre les inéquations.

- $-2x^2 + 3x - 1 \leq 0$

$a < 0$

$$\Delta = b^2 - 4ac = 3^2 - 4 \times -2 \times -1$$

$$= 1$$

$$x_1 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{-3-1}{2 \times -2} = 1$$

$$x_2 = \frac{-b + \sqrt{\Delta}}{2a} = \frac{-3+1}{2 \times -2} = \frac{1}{2}.$$

$\rightarrow S =]-\infty ; \frac{1}{2}] \cup [1 ; +\infty[$

x	$-\infty$		$\frac{1}{2}$		1		$+\infty$
signe		$-$	0	$+$	0	$-$	

• $x^2 - 10x + 25$

$\Delta = b^2 - 4ac = 100 - 4 \times 25 = 0$

$x_0 = -\frac{b}{2a} = \frac{10}{2} = 5.$

x	$-\infty$		5		$+\infty$
signe		$+$	0	$+$	

- $x^2 + 1$

$$\Delta = b^2 - 4ac = 0 - 4 \times 1 \times 1$$
$$= -4 < 0$$

x	$-\infty$	$+\infty$
signe		