Dérivation

exercice :

- $f(x) = x^2 + 1$

  $\hookrightarrow f'(x) = 2x$

- $f(x) = x + \sqrt{x}$

  $\hookrightarrow f'(x) = 1 + \dfrac{1}{2\sqrt{x}}$

- $f(x) = x^3 + x^2$

$\hookrightarrow f'(x) = 3x^2 + 2x$

- $f(x) = x^4 + x + \frac{1}{x}$

$\hookrightarrow f'(x) = 4x^3 + 1 - \frac{1}{x^2}$

- $f(x) = x^2 + x + 4 + \frac{1}{x^2}$

$\hookrightarrow f'(x) = 2x + 1 - \frac{2}{x^3}$.

exercice :

- $f(x) = 3x^2 - 5x$

$\hookrightarrow f'(x) = 3 \times 2x - 5$

$= 6x - 5.$

- $f(x) = \frac{1}{2}x^2 - \frac{1}{3}x + 3$

$\hookrightarrow f'(x) = \frac{1}{2} \times 2x - \frac{1}{3}$

$= x - \frac{1}{3}.$

- $$f(x) = 4x^3 - \frac{2}{3}x^2 + 6$$

$$f'(x) = 4 \times 3x^2 - \frac{2}{3} \times 2x$$
$$= 12x^2 - \frac{4}{3}x$$

- $$f(x) = 3x^5 + 7x^2 + 1$$

$$f'(x) = 3 \times 5x^4 + 14x$$
$$= 15x^4 + 14x$$

exercice :

- $f(x) = \overbrace{x}^{u}\underbrace{\sqrt{x}}_{v}$

$u'(x) = 1$ $\qquad$ $v'(x) = \dfrac{1}{2\sqrt{x}}$.

$$\Rightarrow f'(x) = 1 \times \sqrt{x} + x \times \frac{1}{2\sqrt{x}}$$

$$= \sqrt{x} + \frac{x}{2\sqrt{x}}.$$

- $f(x) = \overbrace{x^2}^{u}\,\overbrace{(2x+4)}^{v}$

$u'(x) = 2x$ $\qquad$ $v'(x) = 2$

$$\Rightarrow f'(x) = 2x(2x+4) + x^2 \times 2$$

$$= 4x^2 + 8x + 2x^2$$

$$= 6x^2 + 8x.$$

- $f(x) = \underbrace{(4x-1)}_{u}\underbrace{(5x+3)}_{v}$

$u'(x) = 4$ $\qquad$ $v'(x) = 5.$

$\Rightarrow f'(x) = 4(5x+3) + 5(4x-1)$

$= 20x + 12 + 20x - 5$

$= 40x + 7$

- $f(x) = \overbrace{(2x+1)}^{u}\, e^{\overbrace{3x+5}^{v}}$

$u'(x) = 2$ $\qquad$ $v'(x) = 3e^{3x+5}$

$$\boxed{(e^u)' = u' e^u}$$

$$\Rightarrow f'(x) = 2e^{3x+5} + (2x+1) \times 3e^{3x+5}$$

$$= e^{3x+5}\left(2 + (2x+1) \times 3\right)$$

$$= e^{3x+5}(2 + 6x + 3)$$

$$= e^{3x+5}(6x + 5).$$

exercice :

- $f(x) = \dfrac{\overbrace{x^4 + 1}^{u}}{\underbrace{x^3 - 1}_{v}}$

$u'(x) = 4x^3$ $\qquad$ $v'(x) = 3x^2$

$$\Rightarrow f'(x) = \frac{4x^3(x^3 - 1) - (x^4 + 1) \times 3x^2}{(x^3 - 1)^2}$$

$$= \frac{4x^6 - 4x^3 - 3x^6 - 3x^2}{(x^3 - 1)^2}$$

$$= \frac{x^6 - 4x^3 - 3x^2}{(x^3 - 1)^2}.$$

$$\bullet\ f(x) = \frac{\overbrace{x^2 - 1}^{u}}{\underbrace{1 - x}_{v}}, \quad x \neq 1$$

$$u'(x) = 2x \qquad v'(x) = -1$$

$$\Rightarrow f'(x) = \frac{2x(1-x) - (x^2-1) \times (-1)}{(1-x)^2}$$

$$= \frac{2x - 2x^2 + x^2 - 1}{(1-x)^2}$$

$$= \frac{-x^2 + 2x - 1}{(1-x)^2 > 0}$$

→ signe ?

$$\Delta = b^2 - 4ac$$

$$= 2^2 - 4 \times -1 \times -1$$

$$= 4 - 4 = 0$$

$$x_0 = -\frac{b}{2a} = -\frac{2}{2 \times -1} = 1$$

x	$-\infty$		1		$+\infty$
signe $-x^2+2x-1$		$-$	0	$-$	

x	$-\infty$		1		$+\infty$
f'		$-$	$\Vert$	$-$	
f		$\searrow$	$\Vert$	$\searrow$	