

Logarithme

- $\ln(a \times b) = \ln(a) + \ln(b)$
- $\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$
- $\ln(a^n) = n \ln(a)$
- $\ln\left(\frac{1}{a}\right) = -\ln(a)$
- $\ln(\sqrt{a}) = \frac{1}{2} \ln(a)$

exercice 1 :

- $\ln\left(\frac{1}{2}\right) = -\ln(2)$
- $\ln(8) = \ln(2 \times 4)$

$$= \ln(2 \times 2 \times 2)$$

$$= \ln(2^3)$$

$$= 3\ln(2).$$

$$\cdot \ln(64) = \ln(8^2)$$

$$= 2\ln(8)$$

$$= 2 \times 3\ln(2)$$

$$= 6\ln(2).$$

$$\ln(e^x) = x = e^{\ln(x)}$$

$$\bullet \ln(2e^2)$$

$$= \ln(2) + \ln(e^2)$$

$$= \ln(2) + 2$$

$$\bullet \ln(64e)$$

$$= \ln(64) + \ln(e^{\textcolor{red}{1}})$$

$$= 6\ln(2) + 1$$

$$\bullet \ln(\sqrt{32})$$

$$= \frac{1}{2} \ln(32)$$

$$= \frac{1}{2} \ln(2 \times 2 \times 2 \times 2 \times 2)$$

$$= \frac{1}{2} \times \ln(2^5)$$

$$= \frac{5}{2} \ln(2).$$

$$\bullet \ln\left(\frac{2}{e}\right)$$

$$= \ln(2) - \ln(e)$$

$$= \ln(2) - 1$$

$$\bullet \ln\left(\frac{32}{e}\right)$$

$$= \ln(32) - \ln(e)$$

$$= 5\ln(2) - 1$$

exercice 2:

$$A = \ln(\sqrt{5} + 2) + \ln(\sqrt{5} - 2)$$

$$= \ln((\sqrt{5} + 2)(\sqrt{5} - 2))$$

$$= \ln((\sqrt{5})^2 - 2^2)$$

$$= \ln(5 - 4)$$

$$= \ln(1)$$

$$= 0$$

$$B = \ln(\sqrt{5} + 2) - \ln(\sqrt{5} - 2)$$

$$= \ln\left(\frac{\sqrt{5} + 2}{\sqrt{5} - 2}\right)$$

$$= P_n \left(\frac{(\sqrt{5}+2) \times (\sqrt{5}+2)}{(\sqrt{5}-2)(\sqrt{5}+2)} \right)$$

$$= P_n \left(\frac{(\sqrt{5}+2)^2}{1} \right)$$

$$= P_n ((\sqrt{5}+2)^2)$$

$$= 2 P_n (\sqrt{5}+2)$$

$$C = P_n (\sqrt{\sqrt{5}+2}) + P_n (\sqrt{\sqrt{5}-2})$$

$$= P_n ((\sqrt{\sqrt{5}+2})(\sqrt{\sqrt{5}-2}))$$

$$= P_n (\sqrt{(\sqrt{5}+2)(\sqrt{5}-2)})$$

$$= P_n (\sqrt{1}) = P_n (1) = 0.$$

$$e^a \begin{matrix} > \\ \leq \end{matrix} e^b \Leftrightarrow a \begin{matrix} > \\ \leq \end{matrix} b$$

$$\ln(a) \begin{matrix} > \\ \leq \end{matrix} \ln(b) \Leftrightarrow a \begin{matrix} > \\ \leq \end{matrix} b$$

$$\ln(x) \begin{matrix} > \\ \leq \end{matrix} a \Leftrightarrow x \begin{matrix} > \\ \leq \end{matrix} e^a$$

exercice 1 :

• $\ln(2x+5) = 0 = \ln(1)$ → $2x+5 > 0$?

$$\Leftrightarrow 2x+5 = 1.$$

$$\Leftrightarrow x = -\frac{4}{2} = -2.$$

$$2x+5 > 0 \Leftrightarrow x > -\frac{5}{2}$$

$$\text{or } -2 > -\frac{5}{2}, \text{ donc } \Delta = \{-2\}$$

$$\cdot \ln(2x-5) = 1 \Rightarrow x > \frac{5}{2} \quad \rightarrow 2x-5 > 0!$$

$$\Leftrightarrow 2x-5 = e$$

$$\Leftrightarrow x = \frac{e+5}{2} > \frac{5}{2}$$

$$\text{done } \lambda = \left\{ \frac{e+5}{2} \right\}$$

$$\cdot \ln(2x-5) = 2 \Rightarrow x > \frac{5}{2} \quad \rightarrow 2x-5 > 0?$$

$$2x-5 = e^2$$

$$x = \frac{e^2+5}{2} > \frac{5}{2}$$

$$\text{done } \lambda = \left\{ \frac{e^2+5}{2} \right\}.$$

$$x-2 > 0? \Rightarrow x > 2$$

$$x-32 > 0? \Rightarrow x > 32$$

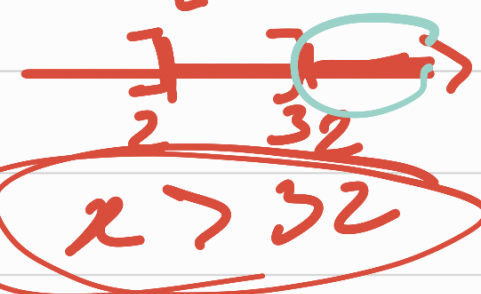
$$\bullet \log(x-2) + \log(x-32) = 6 \log(2)$$

$$\log((x-2)(x-32)) = \log(2^6)$$

$$\Leftrightarrow (x-2)(x-32) = 2^6 = 64$$

$$\Leftrightarrow x^2 - 34x + 64 = 64$$

$$\Leftrightarrow x^2 - 34x + 64 - 64 = 0$$

$$\Leftrightarrow x^2 - 34x = 0$$


A horizontal number line with two points marked: 2 and 32. The region to the right of 32 is circled in green. Below the line, the expression $x > 32$ is written in red and circled in red. A red arrow points from the first equation $x-2 > 0? \Rightarrow x > 2$ to the green circle.

$$\Delta = (-34)^2 - 0 = 1156 > 0$$

$$x_1 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{34 - 34}{2} = 0$$

$$x_2 = \frac{-b + \sqrt{\Delta}}{2a} = \frac{34 + 34}{2} = 34$$

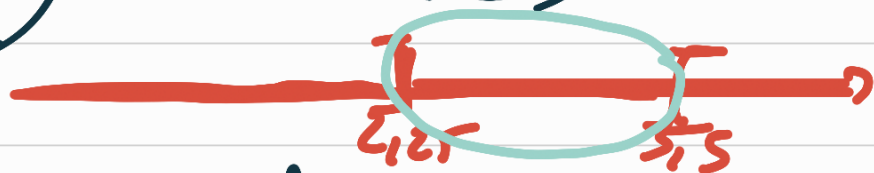
donc $S = \cancel{\{0\}}; \underline{34}$.

$> 0 ? \Rightarrow x < \frac{7}{2}$

$> 0 ? \Rightarrow x > \frac{9}{4}$

• $\ln(-2x+7) - \ln(4x-9) = \ln(3)$

$\ln\left(\frac{-2x+7}{4x-9}\right) = \ln\left(\frac{1}{3}\right)$



$\Leftrightarrow \frac{-2x+7}{4x-9} = \frac{1}{3}$

$\Leftrightarrow 3(-2x+7) = (4x-9) \times 1$

$\Leftrightarrow -6x + 21 = 4x - 9$

$\Leftrightarrow -10x = -30$

$\Rightarrow x = 3.$

