Primitives

exercice 1 :

$$F(x) = (3x^2+5)e^{2x}$$

1) primitive de f est F telle que $F' = f$.

$$F'(x) = 6x e^{2x} + (3x^2+5)\times 2e^{2x}$$

$$= 6x e^{2x} + (6x^2+10)e^{2x}$$

$$= e^{2x}(6x^2+6x+10).$$

$$= f(x)$$

2) Si F est une primitive de f, alors $F + C$ est aussi une primitive de f.

$$F_1(x) = (3x^2+5)e^{2x} + 2.$$

3) $F_2(x) = (3x^2+5)e^{2x} + C$, $C \in \mathbb{R}$

$F_2(0) = 2$

$\Leftrightarrow (3\times 0^2+5)\underbrace{e^0}_{=1} + C = 2$

$\Leftrightarrow \quad 5 + C = 2$

$\Leftrightarrow \quad C = 2 - 5 = -3.$

donc l'unique primitive telle que $F_2(0) = 2$ est :

$$F_2(x) = (3x^2 + 5)e^{2x} - 3$$

exercice 2 : $f(x) = 15$

1) a) $F(x) = 15x + C, \ C \in \mathbb{R}$

b) $F(3) = 12$

$\Leftrightarrow 15 \times 3 + C = 12$

$\Leftrightarrow 45 + C = 12$

$\Leftrightarrow C = 12 - 45 = -33$

donc la primitive est :

$F(x) = 15x - 33$.

2) a) $G(x) = 12 \times \frac{x^2}{2} - 3x + C$, $C \in \mathbb{R}$

$$x^n \xrightarrow{\text{primitive}} \frac{x^{n+1}}{n+1}$$

$G(x) = 6x^2 - 3x + C$, $C \in \mathbb{R}$

b) $G(0) = 3$.

$\Leftrightarrow 0 + C = 3$ donc $G(x) = 6x^2 - 3x + 3$

$\Leftrightarrow C = 3$

exercice 3 :

1) $f(x) = 3x^2 + 6x - 7$

$F(x) = \cancel{3} \times \frac{x^3}{\cancel{3}} + 6\frac{x^2}{2} - 7x$

$= x^3 + 3x^2 - 7x$

4) $f(x) = \frac{x^3}{2} - \frac{5}{x^2} + 7x - 4$

$$\boxed{\frac{1}{x^n} \xrightarrow[n \geq 2]{\text{primitive}} -\frac{1}{n-1} \times \frac{1}{x^{n-1}}}$$

$F(x) = \frac{1}{2} \times \frac{x^4}{4} - 5x - \frac{1}{1} \times \frac{1}{x}$

$+ 7\frac{x^2}{2} - 4x$

$$F(x) = \frac{x^4}{8} + \frac{5}{x} + \frac{7}{2}x^2 - 4x$$

2) $f(x) = 2x^3$

$$F(x) = \frac{2x^4}{4} = \frac{1}{2}x^4$$

3) $f(x) = 7x^5 + 2x^4 - 3x + 8$

$$F(x) = \frac{7x^6}{6} + 2\frac{x^5}{5} - \frac{3x^2}{2} + 8x$$

$$= \frac{7}{6}x^6 + \frac{2}{5}x^5 - \frac{3}{2}x^2 + 8x$$

5)

$$\boxed{\frac{u'}{u^n} \xrightarrow{\text{primitive}} -\frac{1}{n-1} \times \frac{1}{u^{n-1}}}$$

$$f(x) = \frac{6}{(2x+4)^2}$$

$\hookrightarrow u(x)$ $\qquad u'(x) = 2$

$$= 6 \times \frac{1}{2} \times \boxed{\frac{2}{(2x+4)^2}} \qquad = 1 \qquad \frac{u'}{u^2}$$

$$F(x) = 6 \times \frac{1}{2} \times -\frac{1}{2-1} \times \frac{1}{2x+4}$$

$$= 6 \times \frac{1}{2} \times -1 \times \frac{1}{2x+4}$$

$$= -\frac{3}{2x+4}$$

6) $f(x) = \dfrac{2x + 4\times 2}{(x^2+8x-1)^2}$

$\hookrightarrow u(x) \to u'(x) = 2x+8$

$$F(x) = \frac{1}{2} \times -\frac{1}{2-1} \times \frac{1}{x^2+8x-1}$$

$$= -\frac{1}{2} \times \frac{1}{x^2+8x-1}.$$

7) $\boxed{\dfrac{1}{\sqrt{x}} \xrightarrow{\text{primitive}} 2\sqrt{x}}$

$$f(x) = \frac{3}{\sqrt{x}}$$

$$F(x) = 3\times 2\sqrt{x} = 6\sqrt{x}.$$

8) $u'e^u \xrightarrow{\text{primitive}} e^u$

$$f(x) = (-6x+3)e^{x^2-x+5}$$

$\hookrightarrow u$

$u'(x) = 2x-1$

$$= -3\underbrace{(2x-1)}_{u'}\underbrace{e^{x^2-x+5}}_{e^u}$$

$$F(x) = -3\,e^{x^2-x+5}.$$

g)

$$\frac{u'}{\sqrt{u}} \xrightarrow{\text{primitive}} 2\sqrt{u}$$

$$f(x) = \frac{3x^2+1}{\sqrt{x^3+x}}$$

$\hookrightarrow u(x)$

$u'(x) = 3x^2+1$

$$F(x) = 2\sqrt{x^3+x}.$$

exercice 4 :

1) $$f(x) = \underbrace{e^{\overbrace{2x+4}^{u}}}_{\frac{u'}{2} e^{u}}$$

$$F(x) = \frac{1}{2} e^{2x+4} + C \ , \ C \in \mathbb{R}$$

$$F(3) = 5.$$

$$\Leftrightarrow \frac{1}{2} e^{2\times 3+4} + C = 5$$

$$\Leftrightarrow \frac{1}{2} e^{10} + C = 5$$

$$\Leftrightarrow C = 5 - \frac{1}{2} e^{10}$$

Donc $F(x) = \frac{1}{2} e^{2x+4} + 5 - \frac{1}{2} e^{10}.$

2) $\boxed{u'u^n \xrightarrow{\text{primitive}} \dfrac{u^{n+1}}{n+1}}$

$$f(x) = (4x^3 - 10x)(\underbrace{-x^4 + 5x^2 + 10}_{u})^3$$

$\downarrow$

$u'(x) = -4x^3 + 10x$

$$= -(-4x^3 + 10x)(-x^4 + 5x^2 + 10)^3$$

$$F(x) = -\frac{(-x^4 + 5x^2 + 10)^4}{4} + C \quad , C \in \mathbb{R}$$

$$F(2) = 0$$

$$\Leftrightarrow -\frac{(-2^4 + 5 \times 2^2 + 10)^4}{4} + C = 0$$

$$\iff -9604 + C = 0$$

$$\iff C = 9604.$$

donc $F(x) = -\dfrac{(-x^4 + 5x^2 + 10)^4}{4} + 9604.$