TD 2

exercice 10 :

$$4y'' + 4y' + 5y = \sin x \, e^{-x/2}$$

- $4y'' + 4y' + 5y = 0$

$\Rightarrow$ équation caractéristique :

$$4r^2 + 4r + 5 = 0$$

$$\Delta = 16 - 4 \times 4 \times 5 = 16 - 80$$

$$= -64 < 0$$

$$r = \frac{-4 \pm 8i}{8} = -\frac{1}{2} \pm i$$

donc on a :

$$y_h(x) = e^{-x/2}\left(C_1 \cos(x) + C_2 \sin(x)\right)$$

avec $C_1, C_2 \in \mathbb{R}$

• on pose :

$$\boxed{y_p(x) = \cancel{x}\, e^{x/2}\left(A \cos(x) + B \sin(x)\right)}$$

⋮

$$e^{x/2}\left(-8B\sin(x) + 8A\cos(x)\right)$$

$$= \sin e^{x/2}$$

$$\Rightarrow \begin{cases} -8B = 1 \\ 8A = 0 \end{cases} \Rightarrow \begin{cases} B = -1/8 \\ A = 0 \end{cases}$$

donc :

$$y_p(x) = -\frac{e^{x/2}}{8}\sin(x)$$

Donc les solutions :

$$y(x) = y_h(x) + y_p(x).$$

exercice 11 :

$$y'' - 4y' + 4y = d(x)$$

1) $y'' - 4y' + 4y = 0$

=> équation caractéristique :

$$r^2 - 4r + 4 = 0$$

$$\Delta = 16 - 4 \times 4 = 0$$

$$y_h(x) = (C_1 + C_2 x)e^{2x}, \quad C_1, C_2 \in \mathbb{R}$$

2) . $d(x) = e^{2x}$

on cherche $y_{p_1}(x) = Bx^2 e^{2x}$.

- $d(x) = e^{-2x}$

on cherche $y_{p_2}(x) = Be^{-2x}$

3) $d(x) = \dfrac{e^{2x} + e^{-2x}}{4}$

$$y_p(x) = \frac{1}{4} e^{2x} + \frac{1}{4} e^{-2x}$$

$$= \frac{1}{4} y_{p_1}(x) + \frac{1}{4} y_{p_2}(x)$$

Donc :

$$y(x) = y_h(x) + y_p(x).$$

exercice 12 :

$$y'' + y = \underbrace{\frac{1}{\cos x}}_{f(x)} \quad , \quad x \in \left]-\frac{\pi}{2}; \frac{\pi}{2}\right[$$

- $y'' + y = 0 \Rightarrow r^2 + 1 = 0$

$y_h(x) = C_1 \cos(x) + C_2 \sin(x)$,

$C_1, C_2 \in \mathbb{R}$.

- $y_h(x) = C_1 u(x) + C_2 v(x)$

on pose $y_p(x) = C_1(x) u(x) + C_2(x) v(x)$

avec : $\boxed{\begin{cases} C_1' u + C_2' v = 0 \\ C_1' u' + C_2' v' = f(x) \end{cases}}$

$\cdot\ y_h(x) = C_1\cos(x) + C_2\sin(x)$.

$\hookrightarrow$ on pose :

$$y_p(x) = C_1(x)\cos(x) + C_2(x)\sin(x).$$

$$\begin{cases} C_1'(x)\cos(x) + C_2'(x)\sin(x) = 0 \\ -C_1'(x)\sin(x) + C_2'(x)\cos(x) = \dfrac{1}{\cos x} \end{cases}$$

$$\begin{cases} C_1'(x)\sin(x)\cos(x) + C_2'(x)\sin^2(x) = 0 \\ -C_1'(x)\sin(x)\cos(x) + C_2'(x)\cos^2(x) = 1 \end{cases}$$

$$C_1'(x)\sin(x)\cos(x) + C_2'(x)\sin^2(x) = 0$$

$L_2 + L_1 \Rightarrow$ $$C_2'(x)(\sin^2 + \cos^2) = 1$$

$$\Leftrightarrow C_2'(x) = 1$$

$$\Leftrightarrow \boxed{C_2(x) = x}$$

$$\Rightarrow C_1'(x)\sin(x)\cos(x) + 1 \times \sin^2(x) = 0$$

$$\underbrace{\sin(x)}_{\neq 0}\left(C_1'(x)\cos(x) + \sin(x)\right) = 0$$

$$C_1'(x)\cos(x) + \sin(x) = 0$$

$$C_1'(x) = \frac{-\sin(x)}{\cos(x)} \qquad \frac{u'}{u}$$

$$\Leftrightarrow \boxed{C_1(x) = \ln(\cos(x))}$$

donc :

$$y_p(x) = P_n(\cos(x))\cos(x) + x\sin(x)$$

Donc :

$$y(x) = y_h(x) + y_p(x).$$

exercice 5 :

$$y'' + y'^2 = 1.$$

a) on pose $z(x) = e^{y(x)}$

$z' = y'e^y$

$z'' = y''e^y + (y')^2 e^y$

$= e^y \underbrace{(y'' + (y')^2)}_{=1}$

donc $z'' = e^y = z$

$\Rightarrow \quad z'' = z$

$z'' - z = 0$

b) $z(x) = C_1 e^{x} + C_2 e^{-x}, \; C_1, C_2 \in \mathbb{R}$

or $z(x) = e^{y(x)}$

$\Leftrightarrow \; y(x) = \ln(|z(x)|)$

$= \ldots$