

Estimations

→ échantillon : $X_1, \dots, X_n$ iid

→ erreur moyenne quadratique :

$$EQM(\hat{\theta}_n, \theta) = E[(\hat{\theta}_n - \theta)^2].$$

estimateur (pointing to $\hat{\theta}_n$) *paramètre* (pointing to θ)

$$= \underbrace{\text{Var}(\hat{\theta}_n)}_{\text{variance}} - \underbrace{b^2(\hat{\theta}_n, \theta)}_{\text{biais}}$$
$$= E[(\hat{\theta}_n - E[\hat{\theta}_n])^2]$$
$$= E[\hat{\theta}_n^2] - E[\hat{\theta}_n]^2$$

→ biais d'un estimateur :

$$b(\hat{\theta}_n, \theta) = E[\hat{\theta}_n] - \theta.$$

• si $b(\hat{\theta}_n, \theta) = 0$, on dit que l'estimateur est sans-biais

• si $\lim_{n \rightarrow +\infty} b(\hat{\theta}_n, \theta) = 0$, alors on dit que l'estimateur est asymptotiquement sans biais.

→ estimateurs classiques

$$X_1, \dots, X_n \stackrel{\text{iid}}{\hookrightarrow} \mathcal{D}(n; \sigma^2)$$

• un estimateur de la moyenne est :

$$\bar{X}_n = \frac{1}{n} \sum_{i=1}^n X_i$$

• un estimateur de la variance est :

$$\hat{S}_n^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X}_n)^2$$

$$= \frac{1}{n-1} \left(\sum_{i=1}^n X_i^2 - n \bar{X}_n^2 \right)$$

Intervale Confianças

→ intervalo de confiança para
μ (média) com variância conhecida

$$X_1, \dots, X_n \stackrel{\text{iid}}{\sim} N(\mu, \sigma^2)$$

parametro σ^2 → Variança

$$IC_{1-\alpha}(\mu) = \left[\bar{X}_n - z_{1-\frac{\alpha}{2}} \times \frac{\sigma}{\sqrt{n}} ; \bar{X}_n + z_{1-\frac{\alpha}{2}} \times \frac{\sigma}{\sqrt{n}} \right]$$

$1-\alpha$: erro

$$95\% \Rightarrow z_{1-\frac{0.05}{2}} = 1.96$$

$$90\% \Rightarrow z_{1-\frac{0.1}{2}} = 1.64$$

$$99\% \Rightarrow z_{1-\frac{0.01}{2}} = 2.57$$

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Normal

example : 85% confidence.

$$1 - \frac{\alpha}{2} = 1 - \frac{0.15}{2} = 0.925$$

↳ $z = 1.44$

- intervalle de confiance pour la moyenne avec variance inconnue.

$X_1, \dots, X_n \stackrel{\text{iid}}{\sim} \mathcal{N}(\mu, \sigma^2)$
paramètre μ inconnue

$$IC_{1-\alpha}(\mu) = \left[\bar{X}_n - t_{n-1, 1-\frac{\alpha}{2}} \frac{S_n}{\sqrt{n}} ; \bar{X}_n + t_{n-1, 1-\frac{\alpha}{2}} \frac{S_n}{\sqrt{n}} \right].$$

$t_{n-1; 1-\frac{\alpha}{2}}$
degré de liberté.

95% $\Rightarrow 1 - \frac{\alpha}{2} = 0,975$

90% $\Rightarrow 1 - \frac{\alpha}{2} = 0,95$

$$99\% \Rightarrow 1 - \frac{2}{2} = 0,995$$

Student

p	0.7500	0.9000	0.9500	0.9750	0.9900	0.9950	0.9975	0.9990
dl								
1	1.0000	3.0780	6.3140	12.7060	31.8210	63.6570	127.3213	318.3088
2	0.8160	1.8860	2.9200	4.3030	6.9650	9.9250	14.0891	22.3271
3	0.7650	1.6380	2.3530	3.1820	4.5410	5.8410	7.4533	10.2145
4	0.7410	1.5330	2.1320	2.7760	3.7470	4.6040	5.5976	7.1732
5	0.7270	1.4760	2.0150	2.5710	3.3650	4.0320	4.7733	5.8934
6	0.7180	1.4400	1.9430	2.4470	3.1430	3.7070	4.3168	5.2076
7	0.7110	1.4150	1.8950	2.3650	2.9980	3.4990	4.0293	4.7853
8	0.7060	1.3970	1.8600	2.3060	2.8960	3.3550	3.8325	4.5008
9	0.7030	1.3830	1.8330	2.2620	2.8210	3.2500	3.6897	4.2968
10	0.7000	1.3720	1.8120	2.2280	2.7640	3.1690	3.5814	4.1437
11	0.6970	1.3630	1.7960	2.2010	2.7180	3.1060	3.4966	4.0247
12	0.6950	1.3560	1.7820	2.1790	2.6810	3.0550	3.4284	3.9296
13	0.6940	1.3500	1.7710	2.1600	2.6500	3.0120	3.3725	3.8520
14	0.6920	1.3450	1.7610	2.1450	2.6240	2.9770	3.3257	3.7874
15	0.6910	1.3410	1.7530	2.1310	2.6020	2.9470	3.2860	3.7328
16	0.6900	1.3370	1.7460	2.1200	2.5830	2.9210	3.2520	3.6862
17	0.6890	1.3330	1.7400	2.1100	2.5670	2.8980	3.2225	3.6458
18	0.6880	1.3300	1.7340	2.1010	2.5520	2.8780	3.1966	3.6105
19	0.6880	1.3280	1.7290	2.0930	2.5390	2.8610	3.1737	3.5794
20	0.6870	1.3250	1.7250	2.0860	2.5280	2.8450	3.1534	3.5518
21	0.6860	1.3230	1.7210	2.0800	2.5180	2.8310	3.1352	3.5272
22	0.6860	1.3210	1.7170	2.0740	2.5080	2.8190	3.1188	3.5050
23	0.6850	1.3190	1.7140	2.0690	2.5000	2.8070	3.1040	3.4850
24	0.6850	1.3180	1.7110	2.0640	2.4920	2.7970	3.0905	3.4668
25	0.6840	1.3160	1.7080	2.0600	2.4850	2.7870	3.0782	3.4502
26	0.6840	1.3150	1.7060	2.0560	2.4790	2.7790	3.0669	3.4350
27	0.6840	1.3140	1.7030	2.0520	2.4730	2.7710	3.0565	3.4210
28	0.6830	1.3130	1.7010	2.0480	2.4670	2.7630	3.0469	3.4082
29	0.6830	1.3110	1.6990	2.0450	2.4620	2.7560	3.0380	3.3962
30	0.6830	1.3100	1.6970	2.0420	2.4570	2.7500	3.0298	3.3852
35	0.6820	1.3060	1.6900	2.0300	2.4380	2.7240	2.9960	3.3400
40	0.6810	1.3030	1.6840	2.0210	2.4230	2.7040	2.9712	3.3069
45	0.6800	1.3010	1.6790	2.0140	2.4120	2.6900	2.9521	3.2815
50	0.6790	1.2990	1.6760	2.0090	2.4030	2.6780	2.9370	3.2614
100	0.6770	1.2900	1.6600	1.9840	2.3640	2.6260	2.8713	3.1737
inf	0.6745	1.2816	1.6449	1.9600	2.3263	2.5758	2.8070	3.0902

$$\rightarrow ddl = 9 - 1 = 8$$

exemple : 9 individus

et IC_{95%} $\rightarrow 1 - \frac{2}{2} = 0,975$

-> intervalle de confiance pour la variance avec moyenne inconnue

$$X_1, \dots, X_n \xrightarrow{iid} \mathcal{N}(\mu, \sigma^2)$$

μ $\swarrow$ *inconnue* σ^2 $\swarrow$ *paramètre*

$$IC_{1-\alpha}(\sigma^2) = \left[\frac{(n-1)\hat{S}_n}{k_{n-1, 1-\frac{\alpha}{2}}}, \frac{(n-1)\hat{S}_n}{k_{n-1, \frac{\alpha}{2}}} \right]$$

$k_{n-1; 1-\frac{\alpha}{2}}$
 $\hookrightarrow$ degré de liberté.

$\hookrightarrow$ 95% $\Rightarrow 1 - \frac{\alpha}{2} = 0,975$ $\nwarrow 0,025$

90% $\Rightarrow 1 - \frac{\alpha}{2} = 0,95$ $\nwarrow 0,05$

99% $\Rightarrow 1 - \frac{\alpha}{2} = 0,995$ $\nwarrow 0,005$

χ_p^2

khi 2

dl	$\chi_{0.005}^2$	$\chi_{0.01}^2$	$\chi_{0.025}^2$	$\chi_{0.05}^2$	$\chi_{0.1}^2$	$\chi_{0.9}^2$	$\chi_{0.95}^2$	$\chi_{0.975}^2$	$\chi_{0.99}^2$	$\chi_{0.995}^2$
1	.0000	.0002	.0010	.0039	.0158	2.706	3.841	5.024	6.635	7.879
2	.0100	.0201	.0506	.1026	.2107	4.605	5.991	7.378	9.210	10.60
3	.0717	.1148	.2158	.3518	.5844	6.251	7.815	9.348	11.34	12.84
4	.2070	.2971	.4844	.7107	1.064	7.779	9.488	11.14	13.28	14.86
5	.4117	.5543	.8312	1.145	1.610	9.236	11.07	12.83	15.09	16.75
6	.6757	.8721	1.237	1.635	2.204	10.64	12.59	14.45	16.81	18.55
7	.9893	1.239	1.690	2.167	2.833	12.02	14.07	16.01	18.48	20.28
8	1.344	1.646	2.180	2.733	3.490	13.36	15.51	17.53	20.09	21.95
9	1.735	2.088	2.700	3.325	4.168	14.68	16.92	19.02	21.67	23.59
10	2.156	2.558	3.247	3.940	4.865	15.99	18.31	20.48	23.21	25.19
11	2.603	3.053	3.816	4.575	5.578	17.28	19.68	21.92	24.72	26.76
12	3.074	3.571	4.404	5.226	6.304	18.55	21.03	23.34	26.22	28.30
13	3.565	4.107	5.009	5.892	7.042	19.81	22.36	24.74	27.69	29.82
14	4.075	4.660	5.629	6.571	7.790	21.06	23.68	26.12	29.14	31.32
15	4.601	5.229	6.262	7.261	8.547	22.31	25.00	27.49	30.58	32.80
16	5.142	5.812	6.908	7.962	9.312	23.54	26.30	28.85	32.00	34.27
17	5.697	6.408	7.564	8.672	10.09	24.77	27.59	30.19	33.41	35.72
18	6.265	7.015	8.231	9.390	10.86	25.99	28.87	31.53	34.81	37.16
19	6.844	7.633	8.907	10.12	11.65	27.20	30.14	32.85	36.19	38.58
20	7.434	8.260	9.591	10.85	12.44	28.41	31.41	34.17	37.57	40.00
22	8.643	9.542	10.98	12.34	14.04	30.81	33.92	36.78	40.29	42.80
24	9.886	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98	45.56
26	11.16	12.20	13.84	15.38	17.29	35.56	38.89	41.92	45.64	48.29
28	12.46	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28	50.99
30	13.79	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89	53.67
35	17.19	18.51	20.57	22.47	24.80	46.06	49.80	53.20	57.34	60.27
40	20.71	22.16	24.43	26.51	29.05	51.81	55.76	59.34	63.69	66.77
45	24.31	25.90	28.37	30.61	33.35	57.51	61.66	65.41	69.96	73.17
50	27.99	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15	79.49
60	35.53	37.48	40.48	43.19	46.46	74.40	79.08	83.30	88.38	91.95
70	43.28	45.44	48.76	51.74	55.33	85.53	90.53	95.02	100.4	104.2
80	51.17	53.54	57.15	60.39	64.28	96.58	101.9	106.6	112.3	116.3
90	59.20	61.75	65.65	69.13	73.29	107.6	113.1	118.1	124.1	128.3
100	67.33	70.06	74.22	77.93	82.36	118.5	124.3	129.6	135.8	140.2
120	83.89	86.92	91.57	95.70	100.6	140.2	146.6	152.2	159.0	163.6

$$\rightarrow ddf = 8 - 1 = 8$$

exemple : 9 individus

$$\bullet \alpha/2 = 0,005$$

et IC 99% $\rightarrow 1 - \frac{\alpha}{2} = 0,995$