C.C.2 BIS

a) $y'' + ay' + by = 0$

ε, $y(0) = 1 \quad y'(0) = 0$

g) $y_h = A\cos(x) + B\sin(x)$

b) $y'' - 4y' + 4y = e^{2x}$

$y_h(x) = (C_1 + C_2 x)e^{2x}$

on pose $y_p(x) = Ax^2 e^{2x}$

$\Rightarrow A = 1/2$

Donc :

$$y(x) = (c_1 + c_2 x)e^{2x} + \frac{1}{2}x^2 e^{2x}$$

- $y(0) = 0 \iff c_1 = 0$

- $y'(0) = 0 \iff c_2 + 2c_1 = 0$

$$y'(x) = c_2 e^{2x} + (c_1 + c_2 x) \times 2e^{2x} + \frac{1}{2} \times 2x\, e^{2x} + \frac{1}{2}x^2 \times 2e^{2x}$$

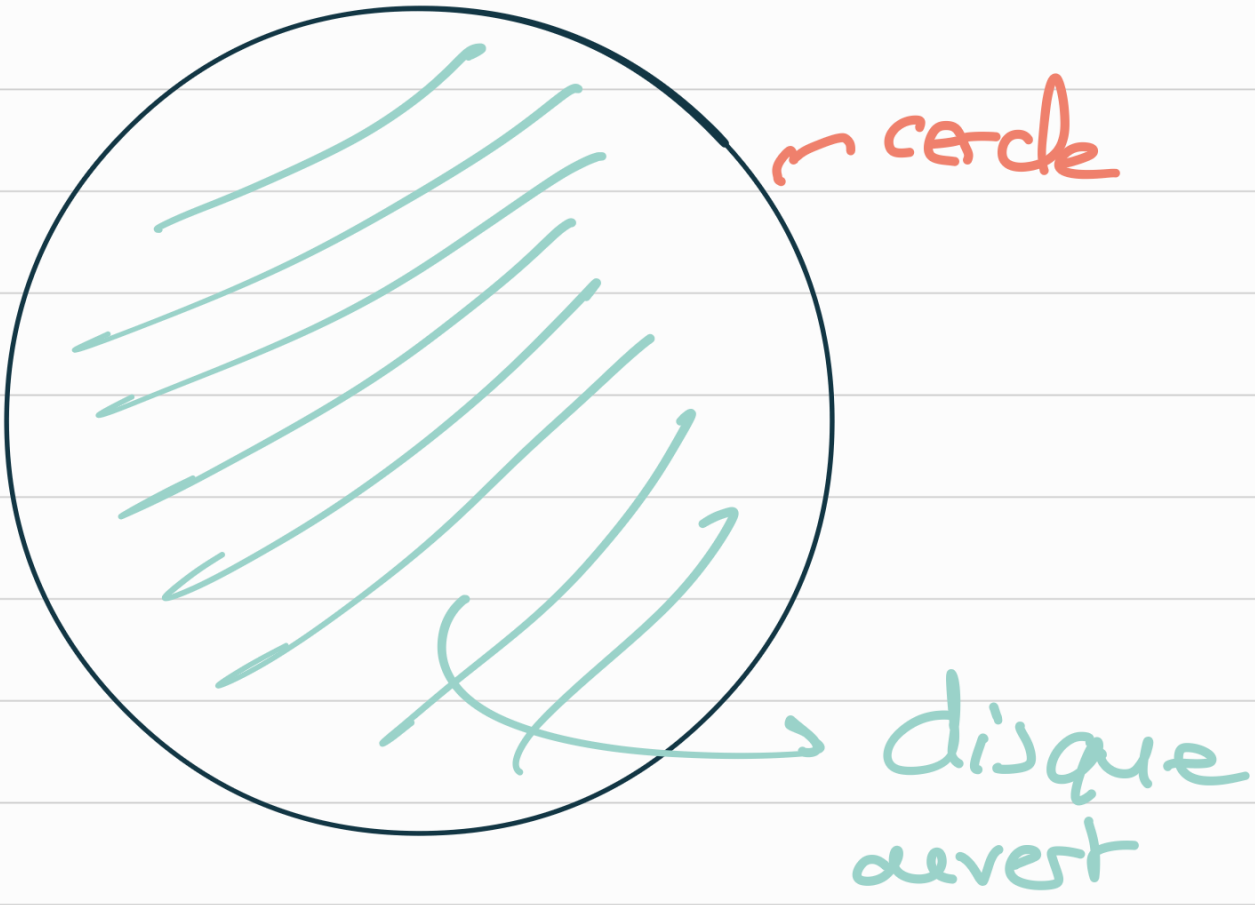
$\Rightarrow c_2 = 0$ car $c_1 = 0$

Donc $y(x) = \frac{x^2}{2} \times e^{2x}$.

b) $1 - x^2 - y^2 > 0$

$$\underline{x^2 + y^2 < 1}$$

↳ disque ouvert



17) $f(r\cos\theta, r\sin\theta)$

$$= \frac{r^2\cos(\theta)\sin\theta}{\sqrt{r^2\cos^2+r^2\sin^2}} = \frac{r^2\cos\theta\sin\theta}{\sqrt{r^2}}$$

$$= r\cos\theta\sin\theta$$

$$0 \leq |f(r\cos\theta, r\sin\theta)| \leq r \times 1 \times 1$$

$$\xrightarrow[r\to 0]{} 0$$

donc $f(r\cos\theta, r\sin\theta) \xrightarrow[r\to 0]{} 0$

$\iff f(x,y) \xrightarrow[(x,y)\to(0,0)]{} 0.$

(a,b) est un point critique si $\nabla f(a,b) = (0,0)$

$$\begin{cases} \dfrac{\partial f}{\partial x}(x,y) = 0 \\ \dfrac{\partial f}{\partial y}(x,y) = 0 \end{cases} \iff \begin{cases} x = a \\ y = b \end{cases}$$

22) $g(x_1; 0) = 0$

$$L_a = \{(x,y) \in \mathbb{R}^2 : f(x,y) = a\}$$

23) $f(x,y) = a$

$x^2 + y^2 = a$ // cercle

$$z = f(x_0, y_0) + \frac{\partial f}{\partial x}(x_0, y_0)(x - x_0) + \frac{\partial f}{\partial y}(x_0, y_0)(y - y_0).$$

$$\frac{\partial f}{\partial x}(x_0, y_0, z_0)(x - x_0) + \frac{\partial f}{\partial y}(x_0, y_0, z_0) \times (y - y_0) + \frac{\partial f}{\partial z}(x_0, y_0, z_0)(z - z_0) = 0$$

25) $\frac{\partial g}{\partial x}(x, y) = 2x + 3y$

$$\frac{\partial g}{\partial y}(x, y) = 3x + 3y^2$$

$$\frac{\partial^2 f}{\partial x \partial y}(x,y) = 3$$

$$\frac{\partial^2 f}{\partial y \partial x}(x,y) = 3$$

$$\frac{\partial^2 f}{\partial x^2}(x,y) = 2$$

$$\frac{\partial^2 f}{\partial y^2}(x,y) = 6y.$$

27) $\frac{\partial f}{\partial x}(x,y) = 4x^3 - 4y$

$\frac{\partial f}{\partial y}(x,y) = 4y^3 - 4x$