TD 4

- **intervalles de confiance sur la moyenne :**

→ **si σ^2 est connue :**

$$IC_{1-\alpha}(\mu) = \left[\overline{X}_n - \underset{\text{normale}}{z_{1-\frac{\alpha}{2}}} \times \frac{\sigma}{\sqrt{n}} \; ; \; \overline{X}_n + z_{1-\frac{\alpha}{2}} \times \frac{\sigma}{\sqrt{n}} \right]$$

→ **si σ^2 est estimée :**

$$IC_{1-\alpha}(\mu) = \left[\overline{X}_n - \underset{\text{student}}{t_{n-1,1-\frac{\alpha}{2}}} \times \frac{S_n}{\sqrt{n}} \; ; \; \overline{X}_n + t_{n-1,1-\frac{\alpha}{2}} \frac{S_n}{\sqrt{n}} \right]$$

- intervalles de confiance sur la variance :

$$IC_{1-\alpha}(\sigma^2) = \left[\frac{(n-1)S_n^2}{\chi^2_{n-1,\,1-\frac{\alpha}{2}}} \; ; \; \frac{(n-1)S_n^2}{\chi^2_{n-1,\,\frac{\alpha}{2}}} \right]$$

khi deux

$$\boxed{S_n^2 = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2}$$

TD 6

• méthode des moments

$X_1, \dots, X_n$ iid

• $m_k = E[X_i^k] = \varphi(\theta)$

• $\hat{m}_{n,k} = \frac{1}{n} \sum_{k=1}^{n} X_i^k$.

$\Rightarrow$ on pose $\hat{\theta}_n = \varphi^{-1}(\hat{m}_{n,k})$

exemple : $X_1, \dots, X_n$ iid $\mathcal{E}(\theta)$

• $m_1 = E[X_1] = \frac{1}{\theta} = \varphi(\theta)$

• $\hat{m}_{n,1} = \frac{1}{n} \sum_{i=1}^{n} X_i =: \overline{X_n}$

on pose $\varphi : x \mapsto \frac{1}{x}$

et $\varphi^{-1} : x \mapsto \frac{1}{x}$

on pose $\vec{\theta}_n = \varphi^{-1}(\vec{m}_{n,1})$

$$= \frac{1}{\overline{x}_n}$$

- **maximum de vraisemblance**

$X_1, \dots, X_n$ iid.

$$\Rightarrow L_x(\theta) = \prod_{i=1}^{n} f_\theta(x_i) \quad \left\{ \prod_\theta (X = x_i) \right.$$

$\Rightarrow$ **la log-vraisemblance**

$$l_x(\theta) = \ln\left(L_x(\theta)\right)$$
$$= \sum_{i=1}^{n} \ln\left(f_\theta(x_i)\right)$$

$\Rightarrow$ le but est de trouver le maximum de $l_x(\theta)$

exemple :

$X_1, \dots, X_n$ iid Loi $\mathcal{B}(\theta)$.

- $f_\theta(x_i) = \mathbb{P}_\theta(X = x_i)$
$$= \theta^{x_i}(1-\theta)^{1-x_i}$$

- $$L_x(\theta) = \prod_{i=1}^{n} f_\theta(x_i)$$
$$= \prod_{i=1}^{n} \theta^{x_i}(1-\theta)^{1-x_i}$$
$$= \theta^{\sum x_i} (1-\theta)^{\sum(1-x_i)}$$
$$= \theta^{\sum x_i} (1-\theta)^{n-\sum x_i}$$
$$= \theta^{n\bar{x}_n} (1-\theta)^{n-n\bar{x}_n}$$

$$\cdot\ \ell_x(\theta) = \ell og\left(L_x(\theta)\right)$$

$$= \ell n\left(\theta^{n\bar{x}_n}(1-\theta)^{n-n\bar{x}_n}\right)$$

$$= \ell n\left(\theta^{n\bar{x}_n}\right) + \ell n\left((1-\theta)^{n-n\bar{x}_n}\right)$$

$$= n\bar{x}_n\,\ell n(\theta) + (n-n\bar{x}_n)\,\ell n(1-\theta)$$

$$\cdot\ \ell_x'(\theta) = \frac{n\bar{x}_n}{\theta} + (n-n\bar{x}_n)\times\frac{-1}{1-\theta}$$

$$\ell_x'(\theta) = 0$$

$$\Longleftrightarrow \frac{n\bar{x}_n}{\theta} - \frac{(n-n\bar{x}_n)}{1-\theta} = 0$$

$$\iff \frac{n\overline{x}_n}{\theta} = \frac{n - n\overline{x}_n}{1-\theta}$$

résoudre

$$\iff \theta = \overline{x}_n$$

- $\ell_x''(\theta) = -\frac{n\overline{x}_n}{\theta^2} - \frac{(n - n\overline{x}_n)}{(1-\theta)^2}$

$$< 0$$

d'où $\ell_x''(\overline{x}_n) < 0$

donc $\theta = \overline{x}_n$ est un maximum global.

Donc $\hat{\theta}_n^{MV} = \overline{x}_n$.

TD 8

- exprimer des données y_i en fonction d'autres données x_i.

$$\hat{Y} = \hat{a} X + \hat{b}$$

$$\boxed{\hat{a} = \frac{\sum_{i=1}^{n} (y_i - \bar{y})(x_i - \bar{x})}{\sum_{i=1}^{n} (x_i - \bar{x})^2} \qquad \hat{b} = \bar{y}_n - \hat{a}\,\bar{x}_n}$$

- **Coefficient de corrélation**

$$\rho = \frac{cov(X,Y)}{\sigma_X \sigma_Y}$$

- **Coefficient de détermination**

$$R^2 = 1 - \frac{\sum (Y_i - \hat{Y}_i)^2}{\sum (Y_i - \bar{Y}_n)}$$