

exercice 1 :

$$\left(\begin{array}{ccc|ccc} -1 & 2 & -1 & 1 & 0 & 0 \\ -3 & 2 & -1 & 0 & 1 & 0 \\ 3 & 0 & 1 & 0 & 0 & 1 \end{array} \right)$$

$$L_1 \rightarrow -L_1 : D_1(-1)$$

$$\left(\begin{array}{ccc} 1 & -2 & 1 \\ -3 & 2 & -1 \\ 3 & 0 & 1 \end{array} \right)$$

$$L_2 \rightarrow L_2 + 3L_1 : T_{21}(3)$$

$$\begin{pmatrix} 1 & -2 & 1 \\ 0 & -4 & 2 \\ 3 & 0 & 1 \end{pmatrix}$$

$$L_3 \rightarrow L_3 - 3L_1 : T_{31}(-3)$$

$$\begin{pmatrix} 1 & -2 & 1 \\ 0 & -4 & 2 \\ 0 & 6 & -2 \end{pmatrix}$$

$$L_2 \rightarrow -\frac{1}{4}L_2 : D_2\left(-\frac{1}{4}\right)$$

$$\begin{pmatrix} 1 & -2 & 1 \\ 0 & 1 & -\frac{1}{2} \\ 0 & 6 & -2 \end{pmatrix}$$

$$L_1 \rightarrow L_1 + 2L_2 : T_{12}(2)$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{2} \\ 0 & 6 & -2 \end{pmatrix}$$

$$L_3 \rightarrow L_3 - 6L_2 : T_{32}(-6)$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{2} \\ 0 & 0 & 1 \end{pmatrix}$$

$$L_2 \rightarrow L_2 + \frac{1}{2}L_3 : T_{23}\left(\frac{1}{2}\right).$$

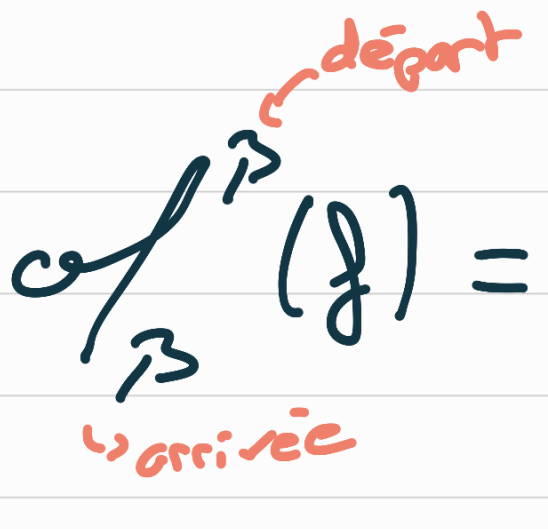
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

done :

$$A^{-1} = T_{23}\left(\frac{1}{2}\right) T_{32}(-6) T_{12}(2) \\ D_2(-\frac{1}{4}) T_{31}(-3) T_{21}(3) \\ D_1(-1).$$

exercice 2 :

1)

 $(f) = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

$\downarrow$ $f(v_1)$ $f(v_2)$ $f(v_3)$

$\rightarrow v_1$ $\rightarrow v_2$ $\rightarrow v_3$

$$f(v_1) = v_2$$

$$f(v_2) = v_3$$

$$f(v_3) = v_1$$

$$2) \mathcal{J}: E \rightarrow E$$

$$\mathcal{M}_{\mathcal{B}}^{\mathcal{B}}(\mathcal{J}) \rightarrow \text{matrix}$$

$$\mathcal{M}_E^E(\mathcal{J}) \rightarrow \text{base canonical}$$

$$P_{E \rightarrow \mathcal{B}} = \begin{pmatrix} -1 & 2 & -1 \\ -3 & 2 & -1 \\ 3 & 0 & 1 \end{pmatrix}$$

$$\begin{aligned} \mathcal{M}_E^E(\mathcal{J}) &= P \mathcal{M}_{\mathcal{B}}^{\mathcal{B}}(\mathcal{J}) P^{-1} \\ &= A \mathcal{M}_{\mathcal{B}}^{\mathcal{B}}(\mathcal{J}) A^{-1} \quad \hookrightarrow \text{exo 1} \\ &= \dots \end{aligned}$$

$$f: E \rightarrow E$$

$$A = \mathcal{M}_E^E(f) \rightarrow \text{canonique}$$

$$B = \mathcal{M}_B^B(f) \rightarrow \text{autre base}$$

$$P = \mathcal{P}_{E \rightarrow B}$$

$$\Rightarrow \begin{cases} B = P^{-1} A P \\ A = P B P^{-1} \end{cases}$$

exercice 3 :

1) $\ker(A) = \{x \in \mathbb{R}^n : Ax = 0_{\mathbb{R}^m}\}$

2) Soit $x \in \ker(A)$

alors $Ax = 0$

on a :

$$\begin{aligned} Cx &= (BA)x \\ &= B(Ax) \\ &= B \times 0 \\ &= 0 \end{aligned}$$

car $x \in \ker(A)$

donc $x \in \ker(C)$

Donc $\ker(A) \subset \ker(C)$.