TD 6

exercice 2 :

2) $$A - 3I_3 = \begin{array}{c} \begin{pmatrix} 0 & a & 0 \\ 0 & 0 & 3 \\ 0 & 0 & 1 \end{pmatrix} \\ \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \end{array}$$

- $a = 0$, $rg(A-3I_3) = 1$

$\Rightarrow \dim(\ker(A-3I_3)) = 2$

- $a \neq 0$, $rg(A-3I_3) = 2$

$\Rightarrow \dim(\ker(A-3I_3)) = 1$

$$A - 4I_3 = \begin{pmatrix} -1 & a & 0 \\ 0 & -1 & b \\ 0 & 0 & 0 \end{pmatrix}$$

$$\sim \begin{pmatrix} -1 & a & ab \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$C_3 + bC_2$

$$\sim \begin{pmatrix} -1 & a & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$C_3 + abC_1$

donc $\forall a, b \in \mathbb{R}$, $\text{rang}(A - 4I_3) = 2$

$\Rightarrow \dim(\text{Ker}(A - 4I_3)) = 1.$

$$A = \begin{pmatrix} 3 & a & 0 \\ 0 & 3 & b \\ 0 & 0 & 4 \end{pmatrix}$$

$$\chi_A(\lambda) = (3-\lambda)^2(4-\lambda)$$

$$\begin{vmatrix} 3-\lambda & a & 0 \\ 0 & 3-\lambda & b \\ 0 & 0 & 4-\lambda \end{vmatrix}$$

• $a \neq 0 \Rightarrow \dim(\underbrace{\ker(A-3I_3)}_{E_3}) = 2$

$\hookrightarrow \dim(E_3) + \dim(E_4) = 2+1$
$= 3$

donc A diagonalisable

• $a = 0 \Rightarrow \dim(\ker(A-3I_3)) = 1$

$\hookrightarrow \dim(E_3) + \dim(E_4) = 1+1$
$= 2 \neq 3$

donc A n'est pas diagonalisable.

3) . $A^2 - 7A + 12I_3 = 0$

→ Initialisation : $n = 0$

$A^0 = I_3$

$3^0 A_3 + 4^0 A_4 = A_3 + A_4$

$= 4I_3 - \cancel{A} + \cancel{A} - 3I_3$

$= 4I_3 - 3I_3$

$= I_3$ ✓

→ **Hérédité** pour un $n \in \mathbb{N}$,

on suppose $A^n = 3^n A_3 + 4^n A_4$

$$A^{n+1} = A^n \times A$$
$$= (3^n A_3 + 4 A_4) A$$
$$= 3^n A_3 A + 4 A_4 A$$

or $A_3 A = (4 I_3 - A) A$
$$= 4A - A^2$$
$$= 4A - 7A + 12 I_3$$
$$= -3A + 12 I_3$$
$$= (4 I_3 - A) \times 3 = 3A_3$$

$$A_4 A = (A - 3I_3) A$$

$$= A^2 - 3A$$

$$= 7A - 12I_3 - 3A$$

$$= 4A - 12I_3$$

$$= (A - 3I_3) \times 4 = 4A_4$$

donc $A^{n+1} = 3^n \times 3A_3 + 4^n \times 4A_4$

$$= 3^{n+1} A_3 + 4^{n+1} A_4$$

✓