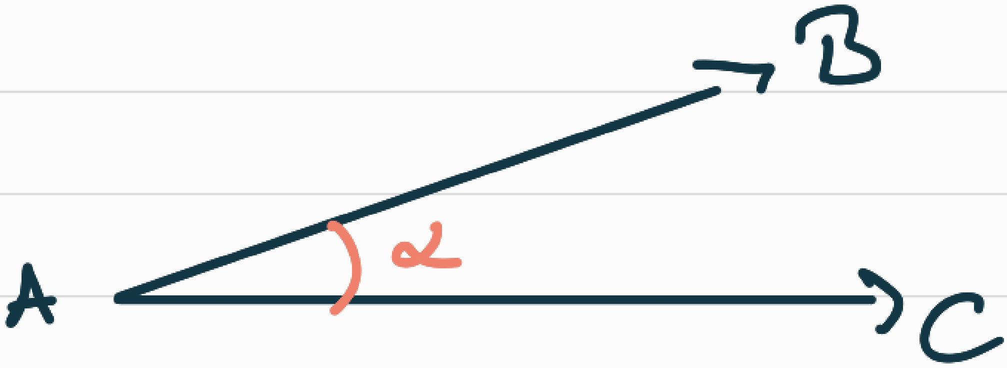
Produit Scalaire



$$\boxed{\overrightarrow{AB} \cdot \overrightarrow{AC} = AB \times AC \times \cos(\widehat{BAC})}$$

- $\overrightarrow{AB} \cdot \overrightarrow{AB} = \|\overrightarrow{AB}\|^2 = AB^2$

- A, B, C trois points

$$\overrightarrow{AB} \cdot \overrightarrow{AC} = \frac{1}{2}\left(AB^2 + AC^2 - BC^2\right)$$

- **propriétés** :

$$\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$$

$$\vec{u} \cdot (\vec{v} + \vec{w}) = \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w}$$

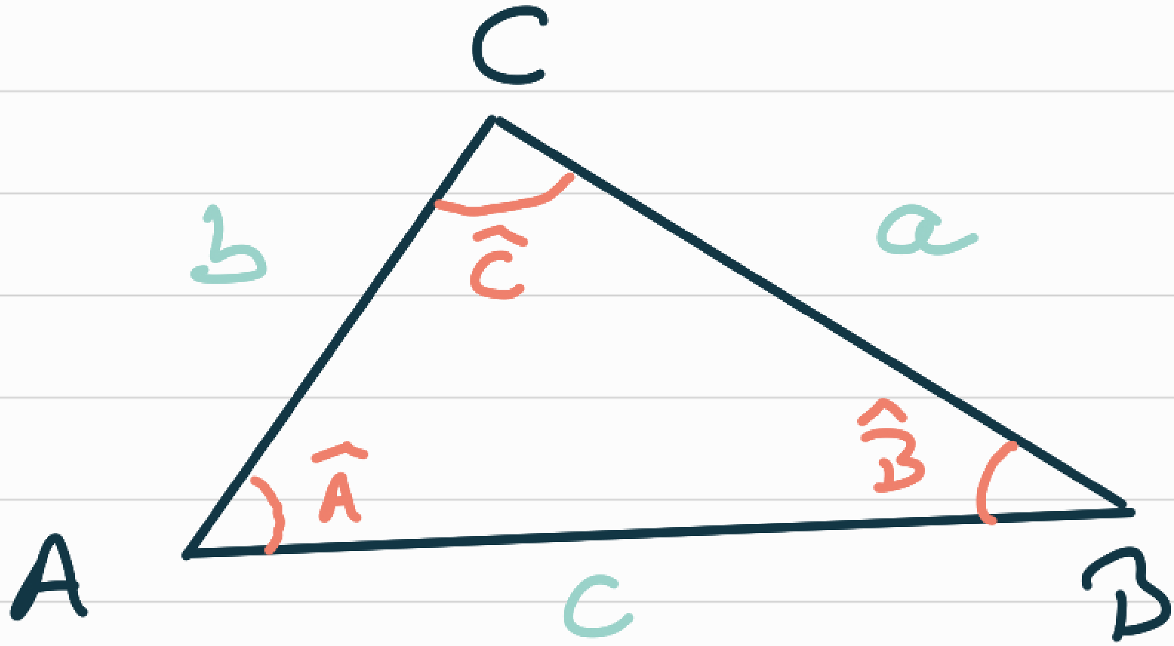
$$\|\vec{u} + \vec{v}\|^2 = \|\vec{u}\|^2 + 2\vec{u} \cdot \vec{v} + \|\vec{v}\|^2$$

$$\|\vec{u} - \vec{v}\|^2 = \|\vec{u}\|^2 - 2\vec{u} \cdot \vec{v} + \|\vec{v}\|^2$$

$$(\vec{u} + \vec{v}) \cdot (\vec{u} - \vec{v}) = \|\vec{u}\|^2 - \|\vec{v}\|^2$$

$$\boxed{\begin{aligned} \vec{u} \cdot \vec{v} &= \frac{1}{2}\left(\|\vec{u} + \vec{v}\|^2 - \|\vec{u}\|^2 - \|\vec{v}\|^2\right) \\ &= \frac{1}{2}\left(\|\vec{u}\|^2 + \|\vec{v}\| - \|\vec{u} - \vec{v}\|^2\right) \end{aligned}}$$

Théorème d'Al Kashi :

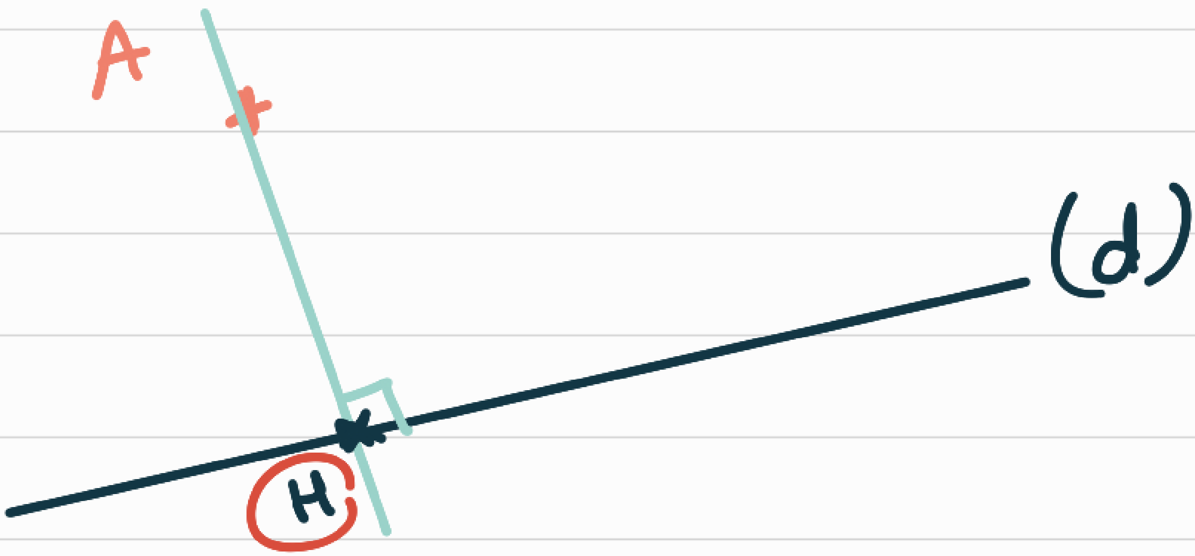


$$\begin{cases} a^2 = b^2 + c^2 - 2bc \times \cos(\hat{A}) \\ b^2 = a^2 + c^2 - 2ac \times \cos(\hat{B}) \\ c^2 = a^2 + b^2 - 2ab \times \cos(\hat{C}) \end{cases}$$

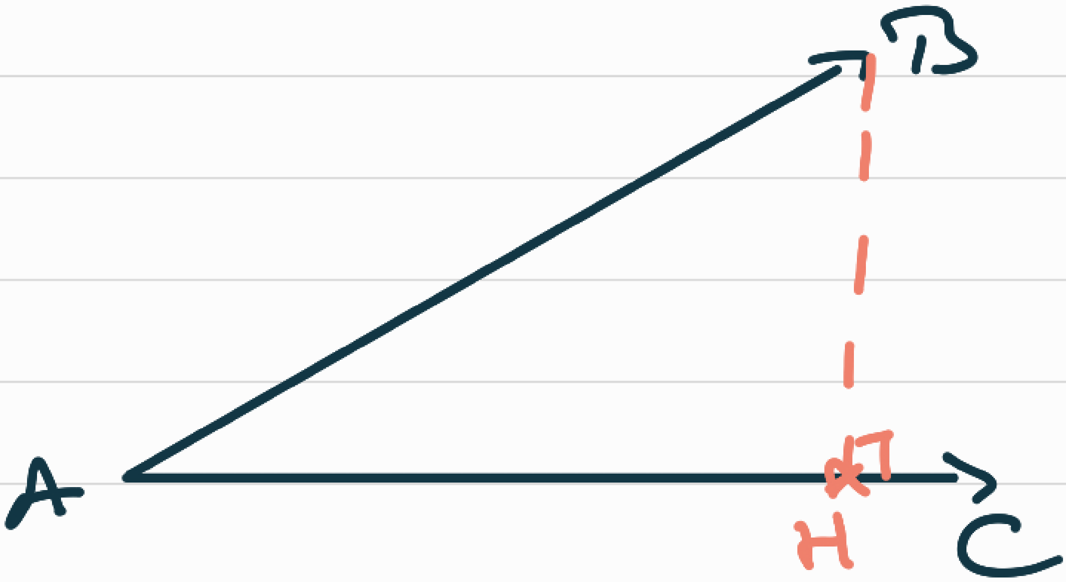
$\vec{u}(x\,;y)$ $\qquad$ $\vec{v}(x'\,;y')$

$$\boxed{\vec{u}\cdot\vec{v} = xx' + yy'}$$

$\vec{u}$ et $\vec{v}$ sont dits orthogonaux si $\vec{u}\cdot\vec{v} = 0$

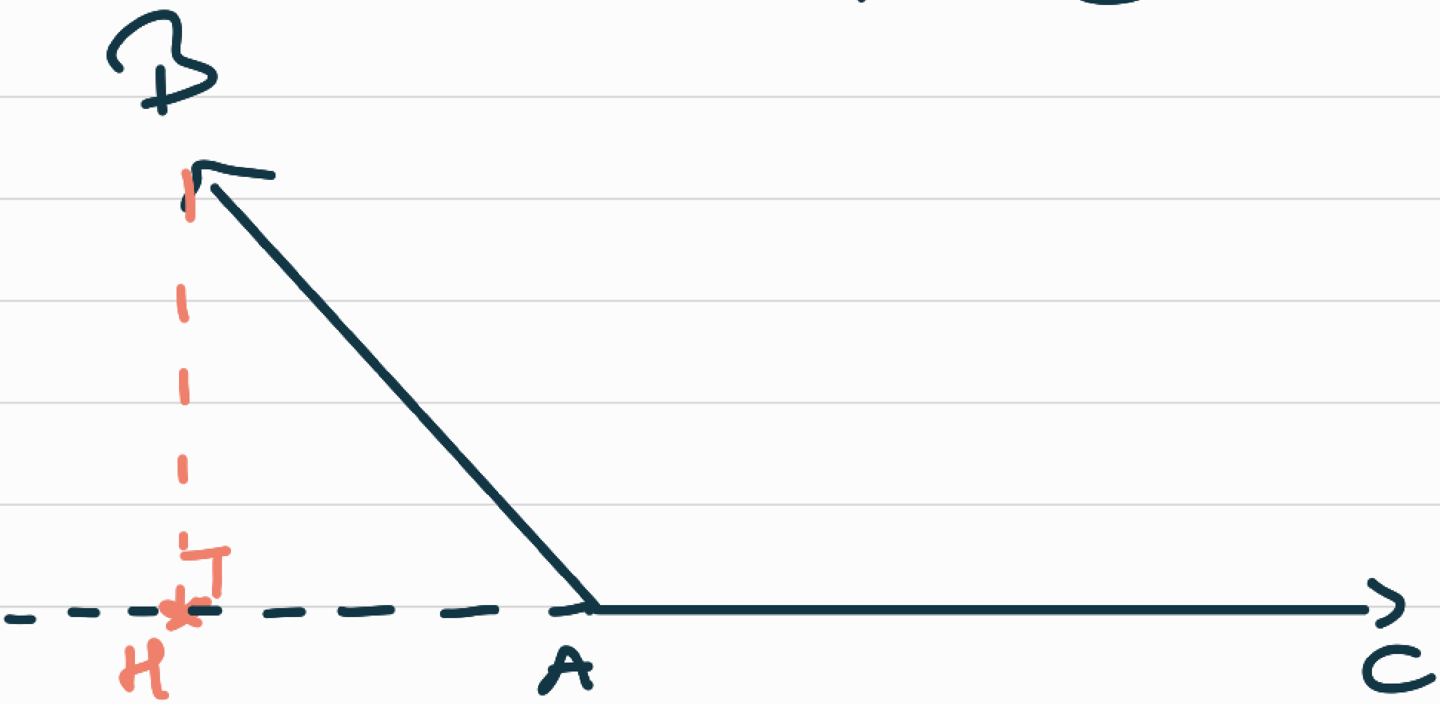


↳ projeté ortho. de A sur (d)



$$\overrightarrow{AB}\cdot\overrightarrow{AC} = \overrightarrow{AH}\cdot\overrightarrow{AC}$$

$$= AH \times AC$$



$$\overrightarrow{AB}\cdot\overrightarrow{AC} = \overrightarrow{AH}\cdot\overrightarrow{AC}$$

$$= -AH \times AC$$

exercice 2 :

a) $\vec{AB} \cdot \vec{AC} = AB \times AC \times \cos(\widehat{BAC})$

$= 6 \times 6 \times \cos(60°)$

$= 6 \times 6 \times \cos\left(\frac{\pi}{3}\right)$

$= 36 \times \frac{1}{2}$

$= 18$

b) $\vec{AB} \cdot \vec{CA} = \vec{AB} \cdot -\vec{AC}$

$= - \vec{AB} \cdot \vec{AC}$

$= -18$

c) $\vec{AC} \cdot \vec{BA}$

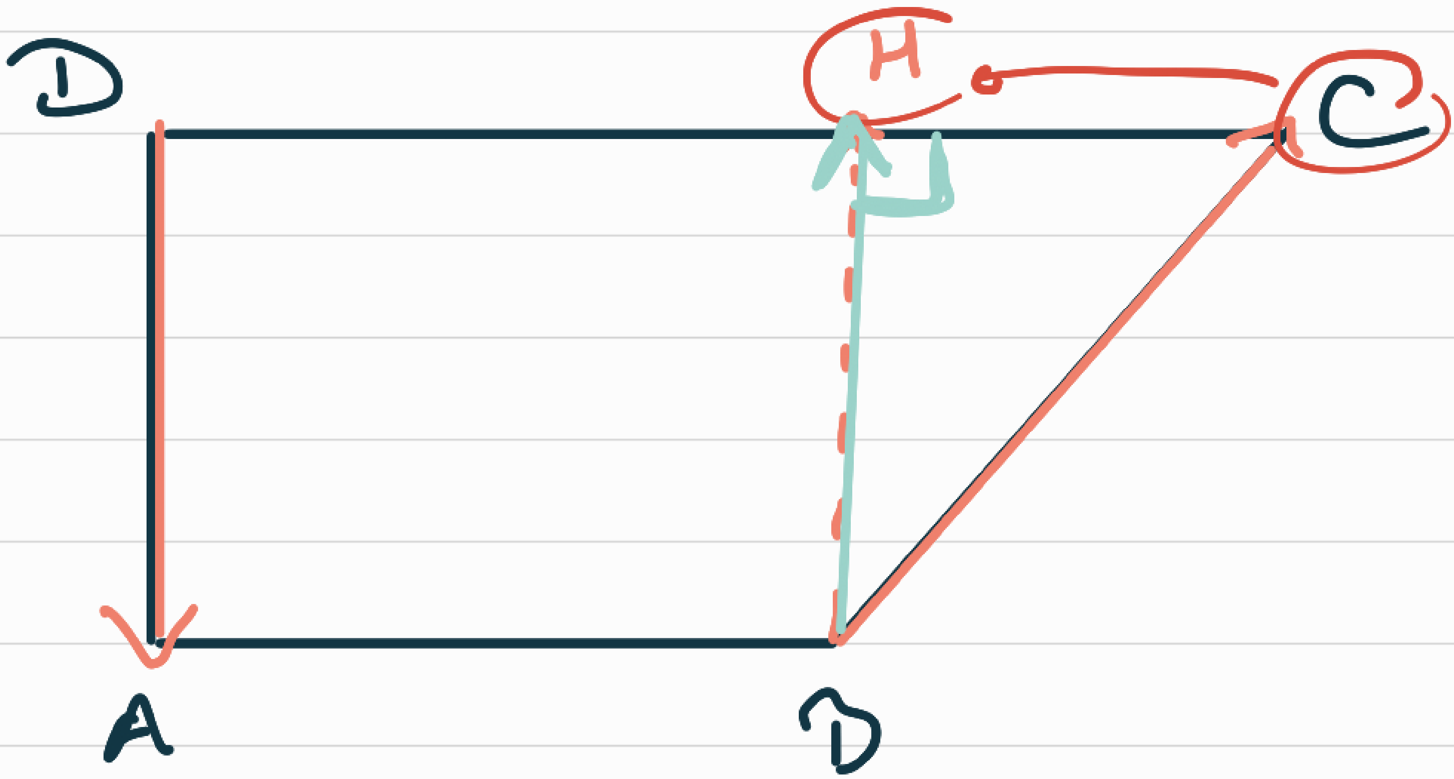
$= \vec{AC} \cdot -\vec{AB}$

$= -\vec{AC} \cdot \vec{AB}$

$= -\vec{AB} \cdot \vec{AC}$

$= -18$

exercice 3 :



a) $\overrightarrow{AD}.\overrightarrow{BC} = ?$

$\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AD} + \overrightarrow{DC}$

$\overrightarrow{AB}.(\overrightarrow{BA} + \overrightarrow{AD} + \overrightarrow{DC})$

$= \overrightarrow{AB}.\overrightarrow{BA} + \overrightarrow{AB}.\overrightarrow{AD} + \overrightarrow{AB}.\overrightarrow{DC}$

$= -\overrightarrow{AB}.\overrightarrow{AB} + 0 + AB \times DC$

$= -5^2 + 5 \times 8$

$= -25 + 40$

$= 15.$

b) $\vec{DC} \cdot \vec{BC}$

$= \vec{DC} \cdot \vec{HC}$

↳ projeté ortho. de B sur (DC)

$= DC \times HC$

$= 8 \times 3$

$= 24$

c) $\vec{BC} \cdot \vec{DA}$

$= \vec{BH} \cdot \vec{DA}$

$= - BH \times DA$

$= - 4 \times 4 = -16.$

exercice 40 :

$A(-3;-1)$ $B(-2;1)$

$C(2;0)$

a) $\vec{AB}.\vec{AC} = ?$

$$\vec{AB}\left(\underbrace{-2-(-3)}_{1} ; \underbrace{1-(-1)}_{2}\right)$$

$$\vec{AC}\left(\underbrace{2-(-3)}_{5} ; \underbrace{0-(-1)}_{1}\right)$$

$$\vec{AB}.\vec{AC} = 1\times5 + 2\times1$$

$$= 5+2 = 7.$$