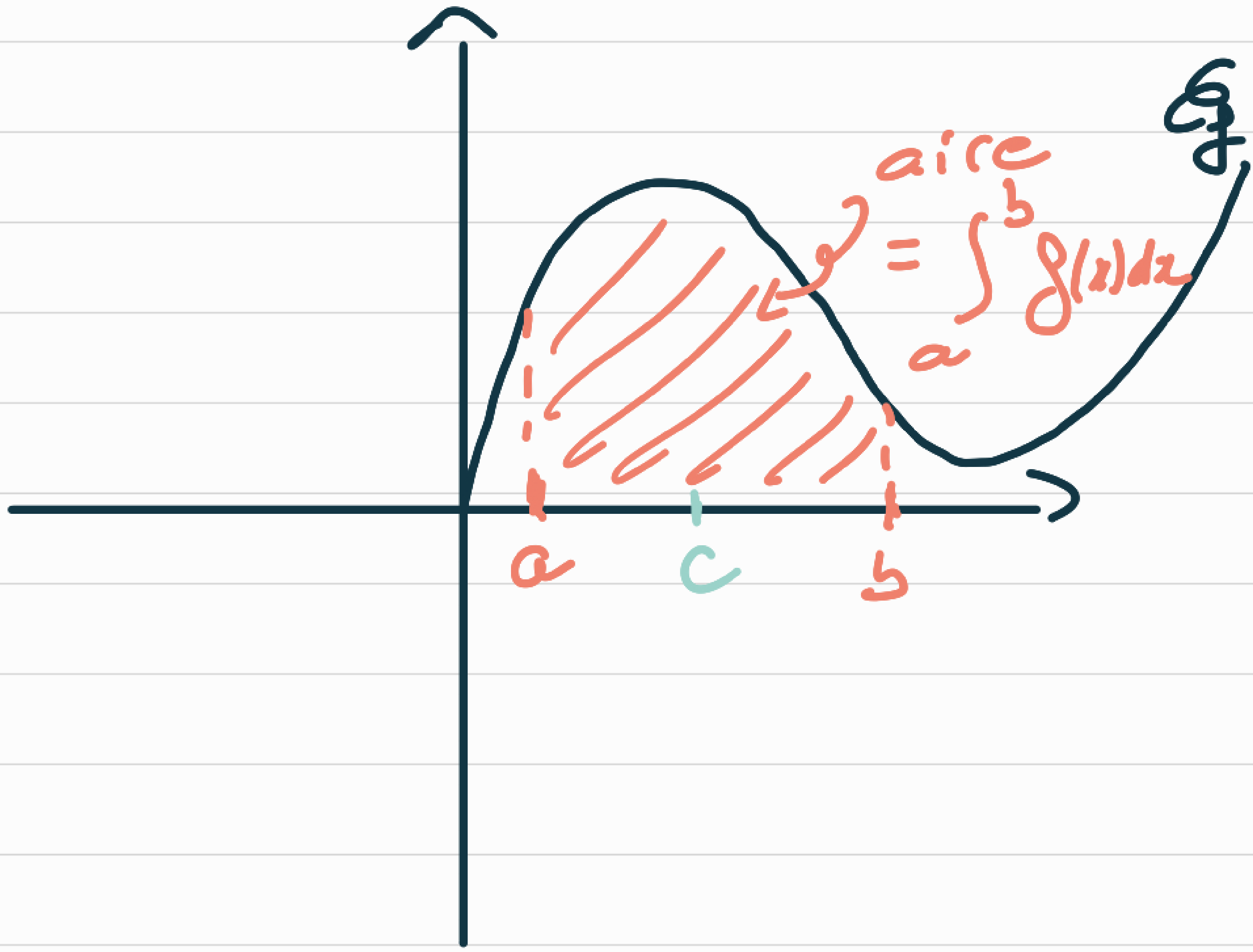
Intégrales



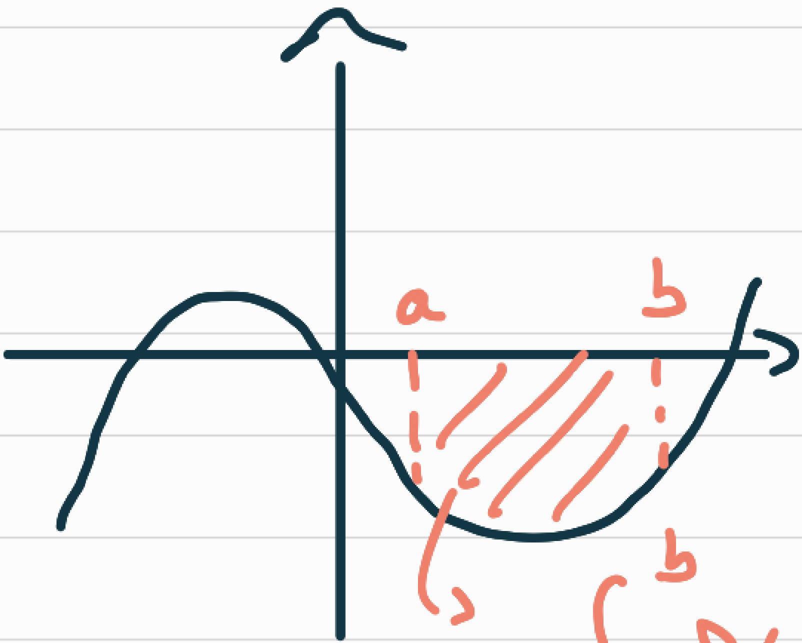
$$\int_a^b f(x)\,dx = \Big[F(x) \Big]_a^b$$

primitive

$$= F(b) - F(a)$$

- $$\int_a^a f(x)\,dx = 0$$

- $$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

- 

$$-\int_a^b f(x)\,dx$$

- $$f \geq g \Rightarrow \int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$$

- $\int_a^b (f+g)(x)\,dx$

$= \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$

- $\int_a^b k f(x)\,dx = k \int_a^b f(x)\,dx$

- <u>valeur moyenne :</u>

$$m = \frac{1}{b-a} \int_a^b f(x)\,dx$$

- **Intégration par parties :**

$$\int_a^b u' \times v \, dx$$

$$= \Big[u \times v \Big]_a^b - \int_a^b u \times v' \, dx$$

$$\int \overset{v}{x} \cdot \overset{u'}{e^{x}} \, dx$$

$$\int \underset{u'}{x} \cdot \overset{v}{\ln(x)} \, dx$$

exercice 1 :

$$f(x) = \underbrace{\int_0^x e^{t^2}\,dt}_{\text{primitive de } e^{t^2}}$$

- $f'(x) = e^{x^2} > 0$

donc f est strictement croissante

exercice 2 :

1) $\displaystyle\int_{-4}^{9} 3 \, dx$

primitive

$= \Big[3x \Big]_{-4}^{9}$

$= (3 \times 9) - (3 \times (-4))$

$= 27 - (-12)$

$= 39$

2) $\int_1^5 (6x^2 + 8x + 2)\, dx$

$$= \left[\frac{\overset{2}{\cancel{6}}x^3}{\cancel{3}} + \frac{\overset{4}{\cancel{8}}x^2}{\cancel{2}} + 2x \right]_1^5$$

$$= (2 \times 5^3 + 4 \times 5^2 + 2 \times 5)$$

$$- (2 \times 1^3 + 4 \times 1^2 + 2 \times 1)$$

$$= 360 - 8$$

$$= 352.$$

3)

$$\int_3^7 \frac{2x}{(x^2+1)^2}\,dx$$

$$\frac{u'}{u^2}$$

$$-\frac{1}{2-1} \times \frac{1}{u^{2-1}}$$

$$= \left[-\frac{1}{2-1} \times \frac{1}{(x^2+1)^{2-1}} \right]_3^7$$

$$= \left[-\frac{1}{x^2+1} \right]_3^7$$

$$= \left(-\frac{1}{7^2+1} \right) - \left(-\frac{1}{3^2+1} \right)$$

$$= \frac{2}{25}.$$

exercice 5 :

1) $\displaystyle\int_0^5 f(x)\,dx$

$$= \int_0^5 \underbrace{(3x^2+6x-9)}_{u'} e^{\overbrace{x^3+3x^2-9x+8}^{e^u}}\,dx \qquad u'e^u \longrightarrow e^u$$

$$= \left[e^{x^3+3x^2-9x+8} \right]_0^5$$

$$= e^{5^3+3\times5^2-9\times5+8} - e^{0^3+3\times0^2-9\times0+8}$$

$$= e^{163} - e^8 > 0$$

2) $f(x) = \underbrace{(3x^2+6x-9)}_{\text{signe}} \underbrace{e^{x^3+3x^2-9x+8}}_{>0}$

l'aire sur $[-2\,;\,2]$

$\mathcal{C}_f$

$f \geqslant 0$ sur $[-2\,;\,2]$

⚠ étudier signe de f sur $[-2\,;\,2]$.

$$\Delta = b^2 - 4ac = 6^2 - 4 \times 3 \times -9$$

$$= 144 > 0$$

$$x_1 = \frac{-b-\sqrt{\Delta}}{2a} = \frac{-6-\sqrt{144}}{2\times 3} = -3$$

$$x_2 = \frac{-b+\sqrt{\Delta}}{2a} = \frac{-6+\sqrt{144}}{2\times 3} = 1$$

x	$-\infty$		-3		-2		1		2		$+\infty$
signe f		$+$	0	$-$		$-$	0	$+$		$+$	

$$\cdot \text{ Aire} = -\int_{-2}^{1} f(x)\,dx + \int_{1}^{2} f(x)\,dx$$

$$= -\left[e^{x^3+3x^2-9x+8}\right]_{-2}^{1}$$

$$+ \left[e^{x^3+3x^2-9x+8}\right]_{1}^{2}$$

$$= -\left(e^{3} - e^{30}\right) + \left(e^{10} - e^{3}\right)$$

$$= -e^{3} + e^{30} + e^{10} - e^{3}$$

$$= e^{30} + e^{10} - 2e^{3}.$$