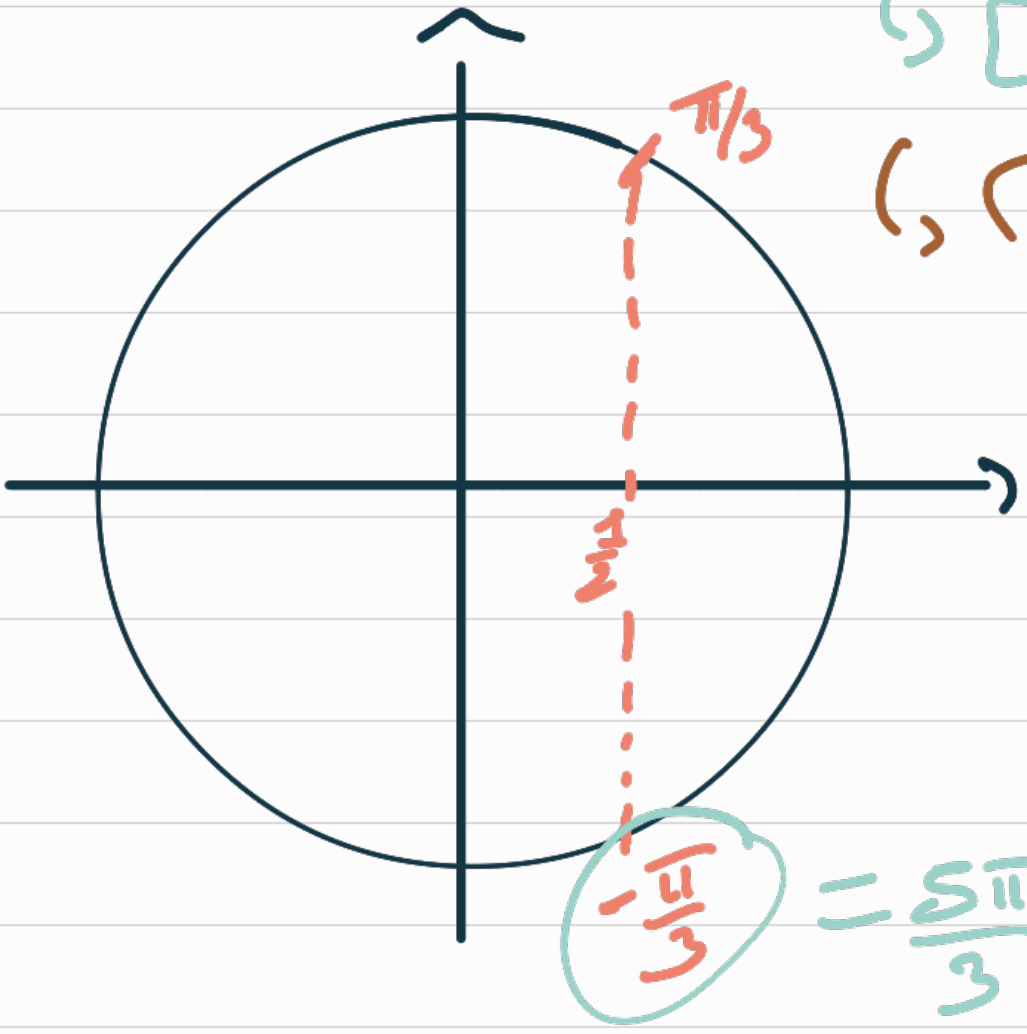
Trigonométrie

- **Equations** :

$\cos(x) = \frac{1}{2}$ sur $]-\pi ; \pi]$

↳ $[0 ; 2\pi[$

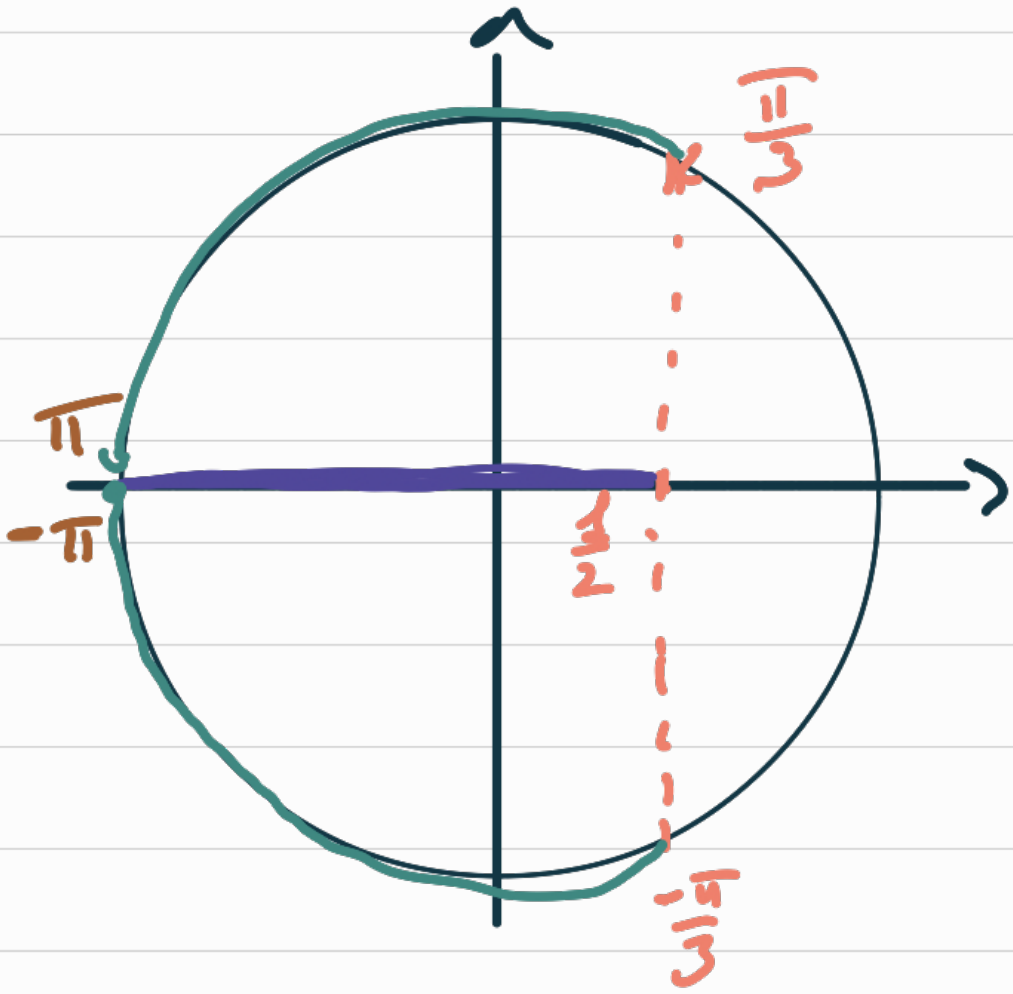
↳ $\mathbb{R}$



$$S = \left\{ \frac{\pi}{3} + 2k\pi ; -\frac{\pi}{3} + 2k\pi \right\}, k \in \mathbb{Z}$$

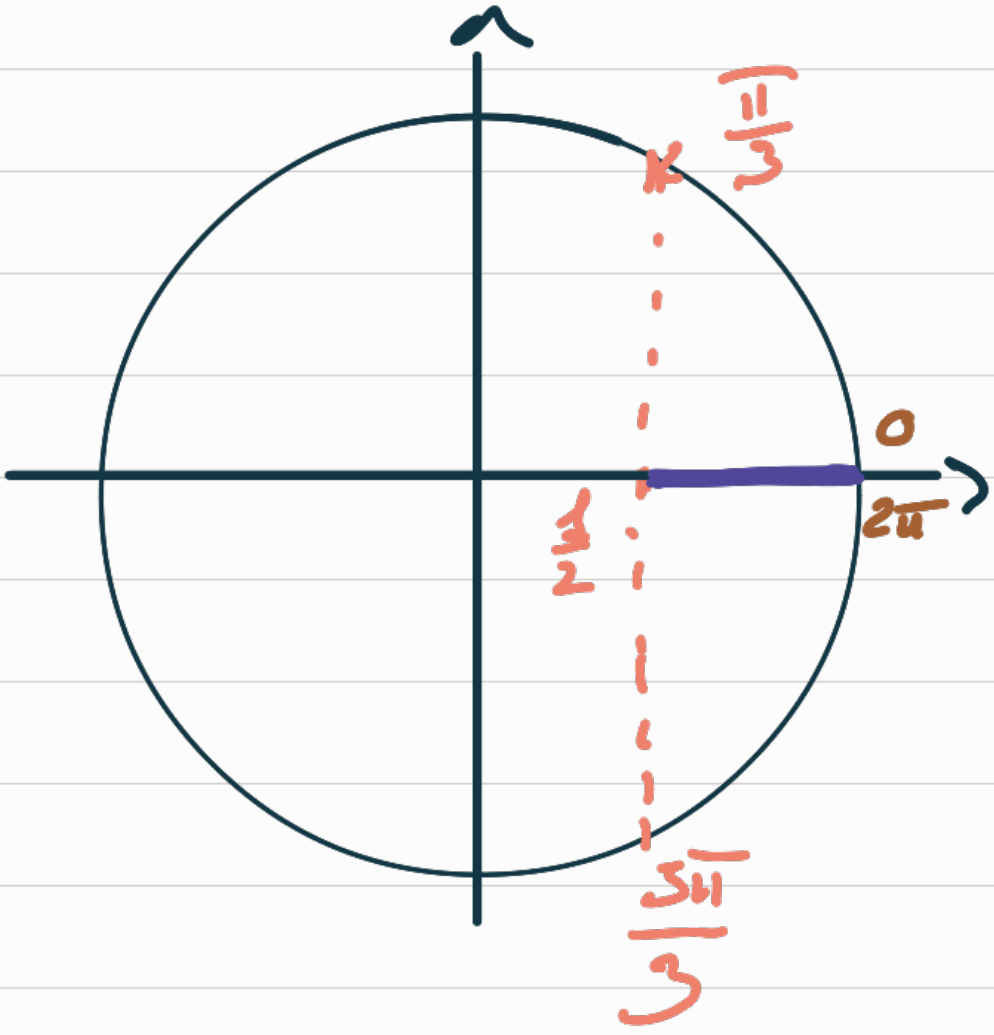
- **inéquations :**

* $\cos(x) \leq \frac{1}{2}$ sur $]-\pi ; \pi]$



$$\mathcal{S} =]-\pi ; -\tfrac{\pi}{3}] \cup [\tfrac{\pi}{3} ; \pi]$$

* $\cos(x) \geq \frac{1}{2}$ sur $[0; 2\pi[$



$\mathcal{S} = [0; \frac{\pi}{3}[\cup]\frac{5\pi}{3}; 2\pi[$.

exercice :

Soit x le réel tel que $\cos(x) = \frac{5}{8}$.

1) Trouver les valeurs possibles de $\sin(x)$

$$\cos^2(x) + \sin^2(x) = 1$$

$$\left(\frac{5}{8}\right)^2 + \sin^2(x) = 1$$

$$\sin^2(x) = 1 - \frac{25}{64}$$

$$= \frac{39}{64}$$

donc $\sin(x) = \frac{\sqrt{39}}{8}$ ou $-\frac{\sqrt{39}}{8}$.

2) Si $x \in [0 ; \frac{\pi}{2}[$, quelle est la valeur de $\sin$?

$$\sin(x) = \frac{\sqrt{39}}{64} > 0 .$$

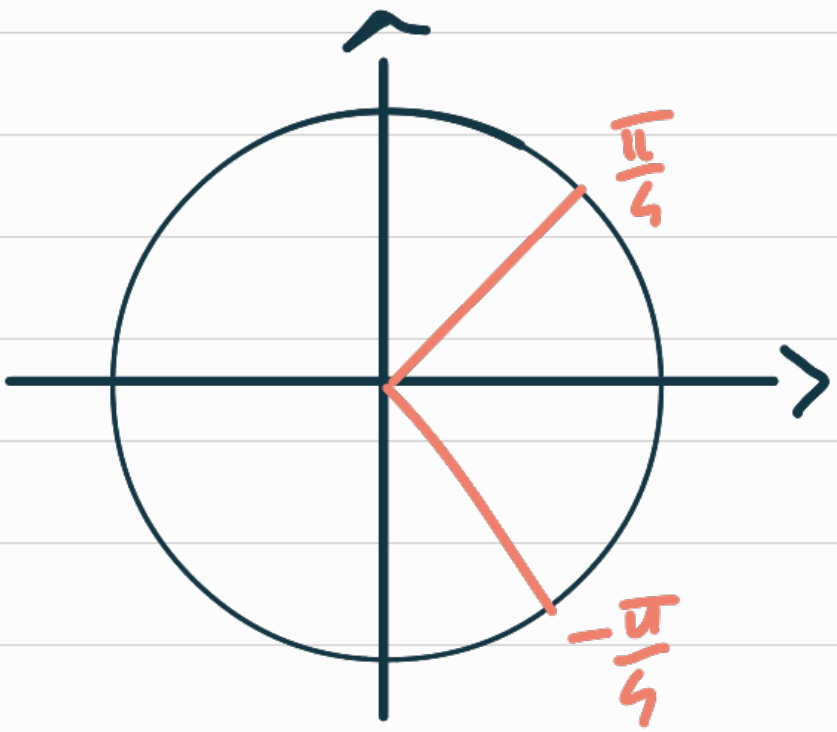
exercice 66 :

$$(2\cos(x) - \sqrt{2})(\sin^2(x) - 1) = 0$$

- $2\cos(x) - \sqrt{2} = 0$

$2\cos(x) = \sqrt{2}$

$\cos(x) = \frac{\sqrt{2}}{2}$.



$S = \{ \frac{\pi}{4} + 2k\pi ; -\frac{\pi}{4} + 2k\pi \text{ / } k \in \mathbb{Z} \}$

. $\sin^2(x) - 1 = 0$

$\sin^2(x) = 1$

$\sin(x) = \pm 1$

$S = \left\{ \frac{\pi}{2} + 2k\pi \; ; \; k \in \mathbb{Z} \right\}$

Donc :

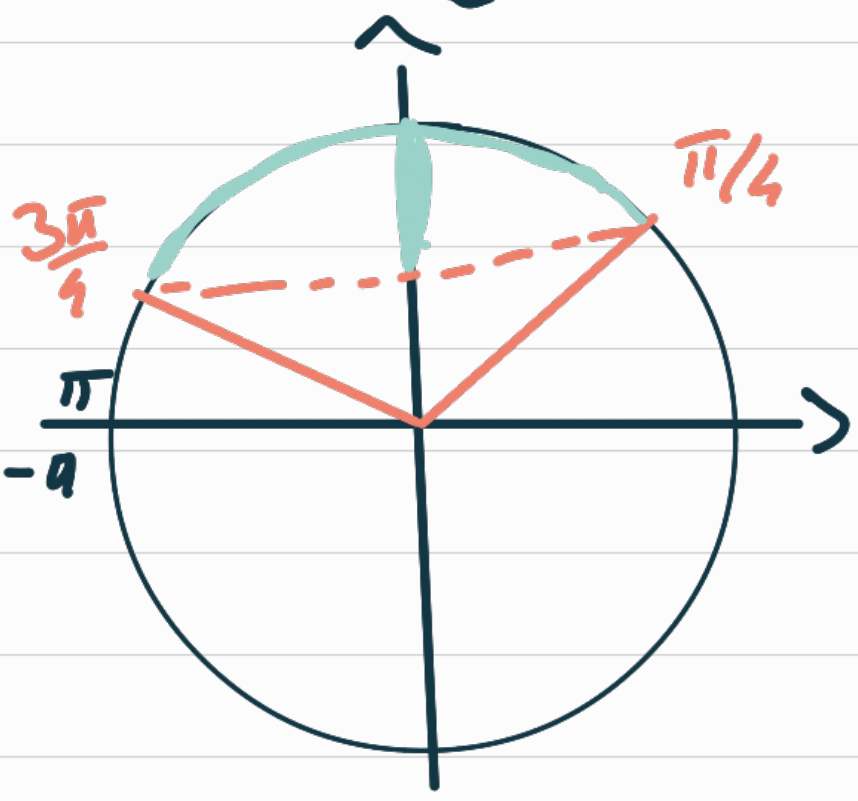
$S = \left\{ \frac{\pi}{2} + k\pi \; ; \; \frac{\pi}{4} + 2k\pi \; ; \right.$

$\left. -\frac{\pi}{4} + 2k\pi \right\}$ avec $k \in \mathbb{Z}$

$$(2\sin(x) - \sqrt{2})(1 - 2\cos(x)) \geq 0$$

• $2\sin(x) - \sqrt{2} \geq 0$ sur $]-\pi;\pi]$

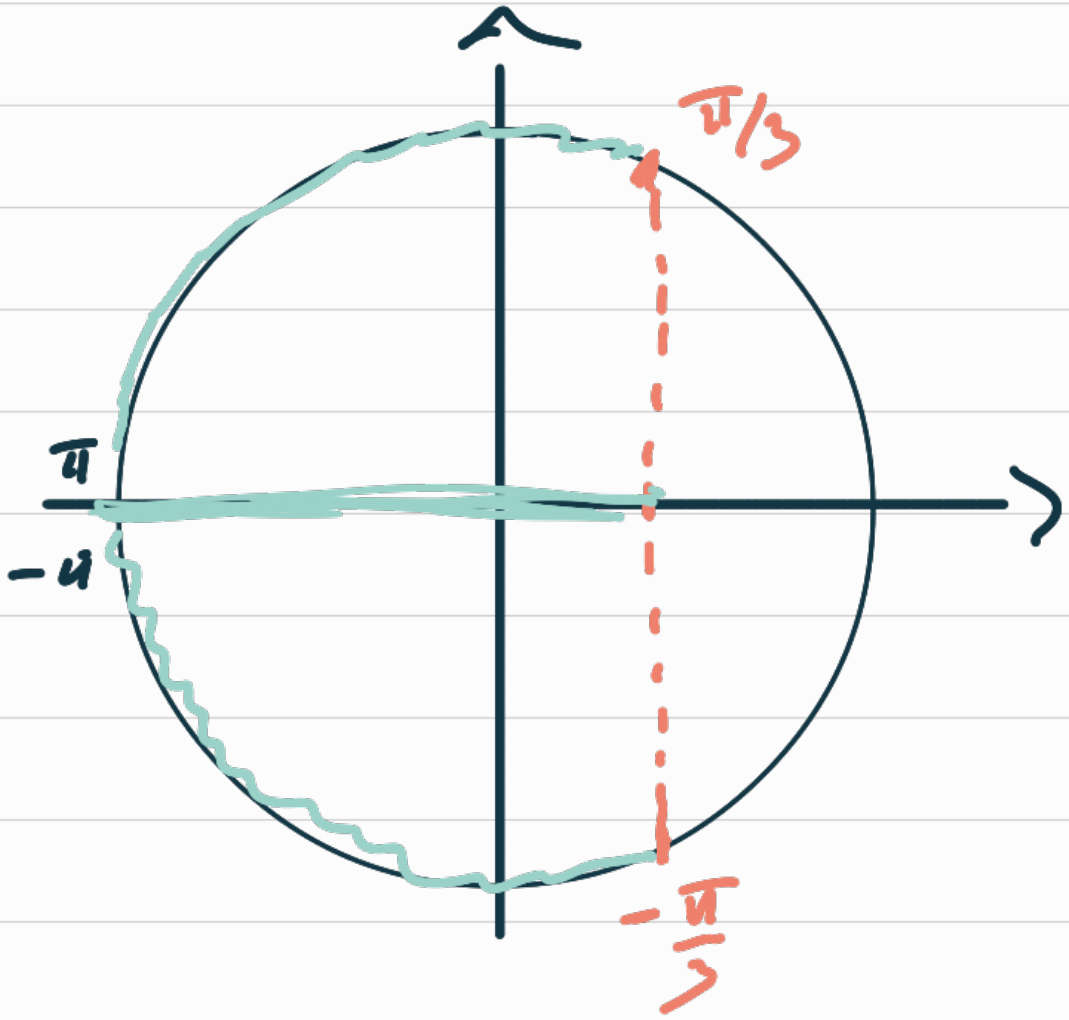
$\Leftrightarrow \sin(x) \geq \frac{\sqrt{2}}{2}$



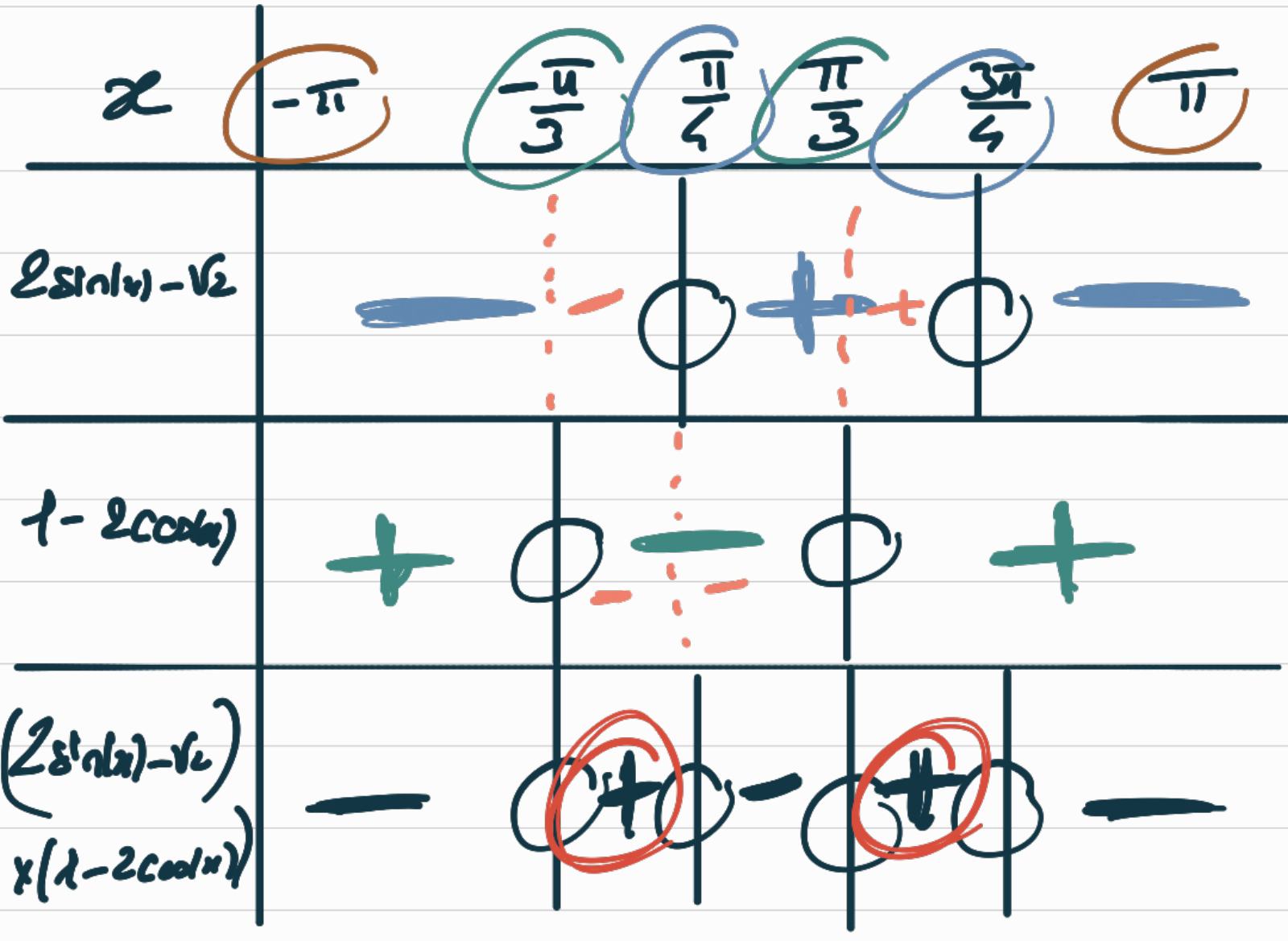
$S = \left[\frac{\pi}{4}; \frac{3\pi}{4}\right]$

. $1 - 2\cos(x) \geq 0$ sur $]-\pi ; \pi]$

$\Leftrightarrow \cos(x) \leq \frac{1}{2}$



$S =]-\pi ; -\frac{\pi}{3}] \cup [\frac{\pi}{3} ; \pi]$



$$S = \left[-\frac{\pi}{3}; \frac{\pi}{4}\right] \cup \left[\frac{\pi}{3}; \frac{3\pi}{4}\right]$$

$$\cos(x)' = -\sin(x)$$

$$\sin(x)' = \cos(x)$$

$$\cos(ax+b)' = -a \times \sin(ax+b)$$

$$\sin(ax+b)' = a \times \cos(ax+b)$$

exercice 56 :

$$f(x) = \frac{1}{\sin(x)} \quad \left(\frac{1}{u}\right)' = \frac{-u'}{u^2}$$

- $D_f = \mathbb{R} \setminus \{ k\pi : k \in \mathbb{Z} \}$

$u(x) = \sin(x)$

$u'(x) = \cos(x)$

$$f'(x) = -\frac{\cos(x)}{\sin(x)^2}$$

exercice 57 :

$$g(x) = \frac{\sin(x)}{\cos(x)}$$

$$D_g = \mathbb{R} \setminus \left\{ \frac{\pi}{2} + k\pi : k \in \mathbb{Z} \right\}$$

$$g'(x) = \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)}$$

$$= \frac{1}{\cos^2(x)}$$

exercice 58 :

$$h(x) = \cos(-2x+4)\sin(-2x+4)$$

$$\mathcal{D}_h = \mathbb{R}$$

$$u(x) = \cos(-2x+4)$$

$$u'(x) = 2\sin(-2x+4)$$

$$v(x) = \sin(-2x+4)$$

$$v'(x) = -2\cos(-2x+4).$$

$$h(x) = 2\sin(-2x+4)\cdot\sin(-2x+4)$$

$$+ \cos(-2x+4)\cdot x - 2\cos(-2x+4)$$

$$= 2\sin^2(-2x+4) - 2\cos^2(-2x+4)$$

$$= 2\left(\sin^2(-2x+4) - \cos^2(-2x+4)\right)$$

exercice 65 :

1) $f(x) = 4\cos(x) + (\cos(x))^2$

$2uu'$ $\leftarrow u^2$

$f'(x) = -4\sin(x) + 2 \times \cos(x) \times -\sin(x)$

$= -2\sin(x)(2 + \cos(x))$

2) . $2 + \cos(x) \geq 0$

$\iff \cos(x) \geq -2$

or $-1 \leq \cos(x) \leq 1$

donc $-2 \leq -1 \leq \cos(x)$

• $-2\sin(x) \geq 0 \iff \sin(x) \leq 0$

x	0		π		2π
$-2\sin(x)$	$-$	$-$	0	$+$	$+$
$2+\cos x$	$+$	$+$	$+$	$+$	$+$
f'	$-$	$-$	0	$+$	$+$
f	5	↘	-3	↗	5