SL Final

exercice 2 :

partie linéaire

$$\boxed{A x^2 + 1 y^2 - 4xy} \boxed{-2x - 2y} + 1 = 0$$

quadratic part

$$M = \begin{pmatrix} A & \frac{C}{2} \\ \frac{C}{2} & B \end{pmatrix} = \begin{pmatrix} 1 & -2 \\ -2 & 1 \end{pmatrix}$$

↳ M est symétrique à valeurs réelles donc elle est diagonalisable dans $\mathbb{R}$ par $M = PDP^T$.

$$P_A(\lambda) = \begin{vmatrix} 1-\lambda & -2 \\ -2 & 1-\lambda \end{vmatrix} = (1-\lambda)^2 - 4$$

$$= \lambda^2 - 2\lambda + 1 - 4$$

$$= \lambda^2 - 2\lambda - 3$$

$$= (\lambda + 1)(\lambda - 3)$$

donc M a pour valeur propre $\lambda_1 = -1$ et $\lambda_2 = 3$

- positive eigenvalues : ellipse
- one zero eigenvalue : parabola
- eigenvalues of different signs : hyperbola

La conique est une hyperbole.

But : trouver le centre et trouver $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

- $E_{-1} = \ker(M + I)$
$= \text{Span}\{(1;1)\}$

- $E_3 = \ker(M - 3I)$
$= \text{Span}\{(-1;1)\}$

$$e_1 = \frac{1}{\sqrt{2}}(1,1)$$

$$e_2 = \frac{1}{\sqrt{2}}(-1;1).$$

on pose :

$$P = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

et on note $\overset{\begin{pmatrix} u \\ v \end{pmatrix}}{\tilde{X}} = P^T \overset{\begin{pmatrix} x \\ y \end{pmatrix}}{X}$

avec $P^T = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$

$$\Rightarrow \begin{cases} u = \dfrac{x - y}{\sqrt{2}} \\[2ex] v = \dfrac{x + y}{\sqrt{2}} \end{cases}$$

- **Linear part**: $-2x-2y$

$$v = \frac{x+y}{\sqrt{2}}$$

$$-v\sqrt{2} = -x-y$$

$$\boxed{-2\sqrt{2}\,v} = -2x-2y.$$

- **quadratic part**:

$$x^2+y^2-5xy$$

$$= (\tilde{X})^T \times D \times \tilde{X}$$

$$= (u\ \ v)\begin{pmatrix} 3 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix}$$

$$= 3u^2 - v^2$$

$$x^2 + y^2 - 4xy - 2x - 2y + 1 = 0$$

$$3u^2 \underbrace{- v^2 - 2\sqrt{2}\, v}_{} + 1 = 0$$

$$-\left(v^2 + 2\sqrt{2}\, v\right)$$

$$= -\left[\left(v + \sqrt{2}\right)^2 - 2\right]$$

$$3u^2 - \left[\left(v + \sqrt{2}\right)^2 - 2\right] + 1 = 0$$

$$3u^2 - \left(v + \sqrt{2}\right)^2 + 2 + 1 = 0$$

$$3u^2 - \left(v + \sqrt{2}\right)^2 + 3 = 0$$

$$3u^2 - \left(v + \sqrt{2}\right)^2 = -3$$

$$(v+\sqrt{2})^2 - 3u^2 = 3$$

$$\boxed{\frac{(v+\sqrt{2})^2}{3} - \frac{u^2}{1} = 1.}$$

v_0

$3 \to a^2$

$1, 3^2$

$\Rightarrow$ centre $(0\,;\,-\sqrt{2})$

Coordonnées (u,v)

$$\begin{cases} u = 0 = \dfrac{x-y}{\sqrt{2}} \\ v = -\sqrt{2} = \dfrac{x+y}{\sqrt{2}} \end{cases}$$

$$\begin{cases} \dfrac{x-y}{\sqrt{2}} = 0 \\ \dfrac{x+y}{\sqrt{2}} = -\sqrt{2} \end{cases} \Longleftrightarrow \begin{cases} x = y \\ x+y = -2 \end{cases}$$

$$\begin{cases} x = y \\ 2x = -2 \end{cases} \iff \begin{cases} y = -1 \\ x = -1 \end{cases}$$

donc $\boxed{\text{centre } (-1\,;-1)}$

- $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$

$\Rightarrow$ asymptote : $y = \pm\left(\frac{b}{a}\right)x$

$$\frac{(v+\sqrt{2})^2}{3} - u^2 = 1$$

$$\iff \frac{V^2}{3} - u^2 = 1$$

$\Rightarrow$ asymptote : $u = \pm \frac{1}{\sqrt{3}} v$

$= \pm \frac{1}{\sqrt{3}} (v + \sqrt{2})$

$$\boxed{\sqrt{3}\, u = \pm (v + \sqrt{2})}$$

Coordinates (u, v)

$$\begin{cases} u = \dfrac{x - y}{\sqrt{2}} \\ v = \dfrac{x + y}{\sqrt{2}} \end{cases}$$

$$\sqrt{3} \times \frac{(x - y)}{\sqrt{2}} = \pm \left(\frac{x+y}{\sqrt{2}} + \sqrt{2} \right)$$

$\times \sqrt{2}$ $\times \sqrt{2}$

$$\sqrt{3}\,(x - y) = \pm (x + y + 2)$$

$$\begin{cases} x + y + 2 = \sqrt{3}(x - y) & (1) \\ x + y + 2 = -\sqrt{3}(x - y) & (2) \end{cases}$$