exercice 1 :

2) $u_{n+1} - u_n$

$= u_n + 2n - 11 - u_n$

$= 2n - 11$

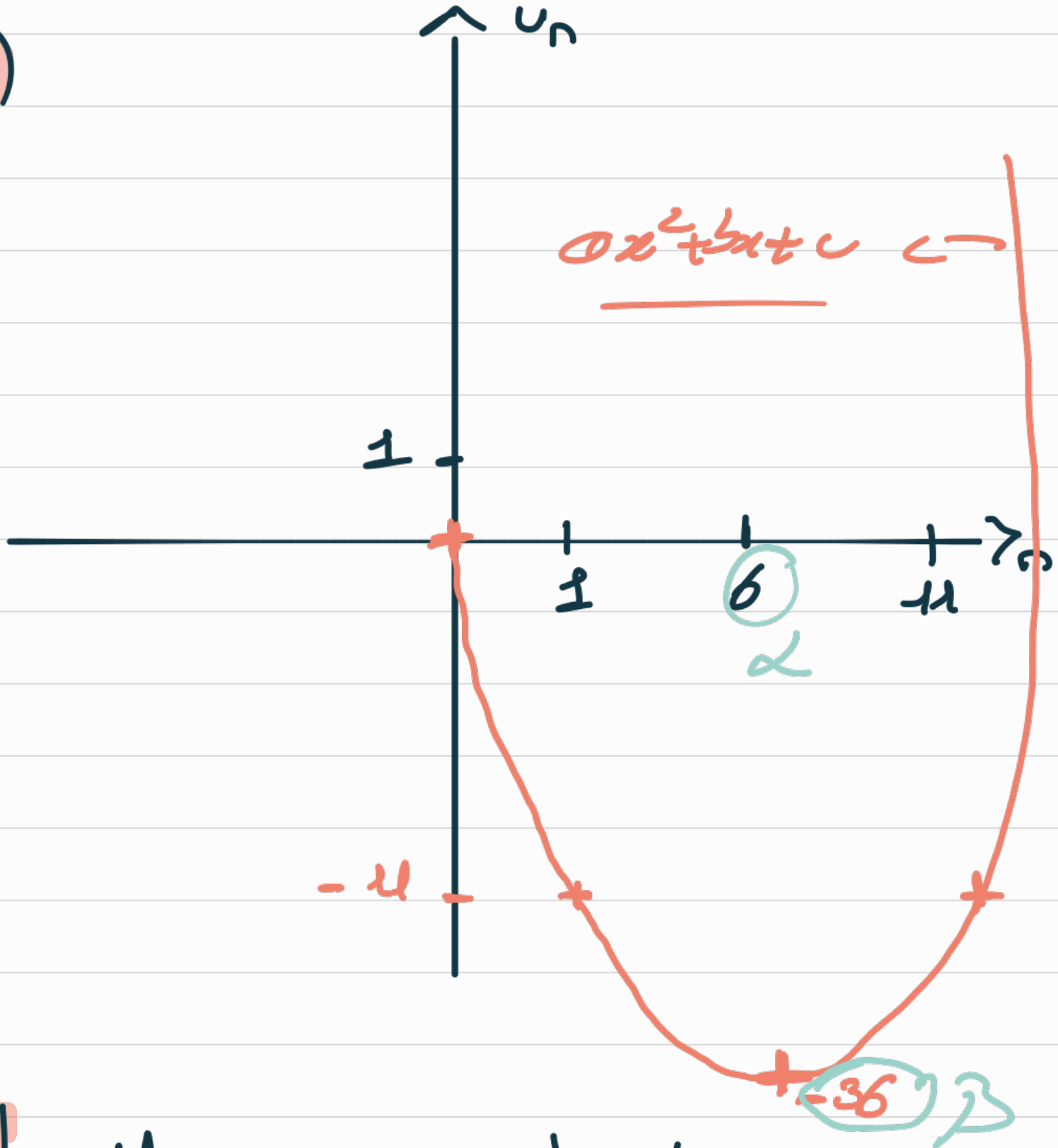
• $2n - 11 \geq 0$

$\iff 2n \geq 11$

$\iff n \geq \frac{11}{2} = 5,5$

donc $u_{n+1} - u_n \geq 0$ à partir du rang 6, donc (u_n) est croissante à partir du rang 6.

3)



4) $u_0 = 0$ et $u_{11} = 0$

$$\Rightarrow \boxed{u_n = (n-6)^2 - 36}$$

$$(x-\alpha)^2 + \beta$$

5) $f(x) = x^2 - 12x$

$ax + b$

a) $f'(x) = 2x - 12$

> 0

$-\frac{b}{a} = -\frac{(-12)}{2}$

x	$-\infty$		6		$+\infty$
f'		$-$	0	$+$	
f		↘	-36	↗	

$f(6)$

b) $u_n = f(n)$.

f est croissante sur $[6; +\infty[$

donc (u_n) est croissante à partir du rang 6.

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exercice 2 :

1) $AB = \sqrt{(-1)^2 + 5^2}$

$= \sqrt{26}$.

2) $\vec{AB} \cdot \boxed{\vec{CD}}$ ← $-\vec{BC}\begin{pmatrix}1\\-5\end{pmatrix}$

$= -4 \times 1 + 3 \times (-5)$

$= -19$

3) $\begin{cases} \vec{u} \cdot \vec{v} = 0 \\ \|\vec{u}\| = 3 \ , \ \|\vec{v}\| = 2. \end{cases}$

$$(\vec{u}+\vec{v}).(\vec{u}-\vec{v}) = \|\vec{u}\|^2 - \|\vec{v}\|^2$$

$$(a+b)(a-b) = a^2 - b^2$$

$$= 3^2 - 2^2$$

$$= 9 - 4$$

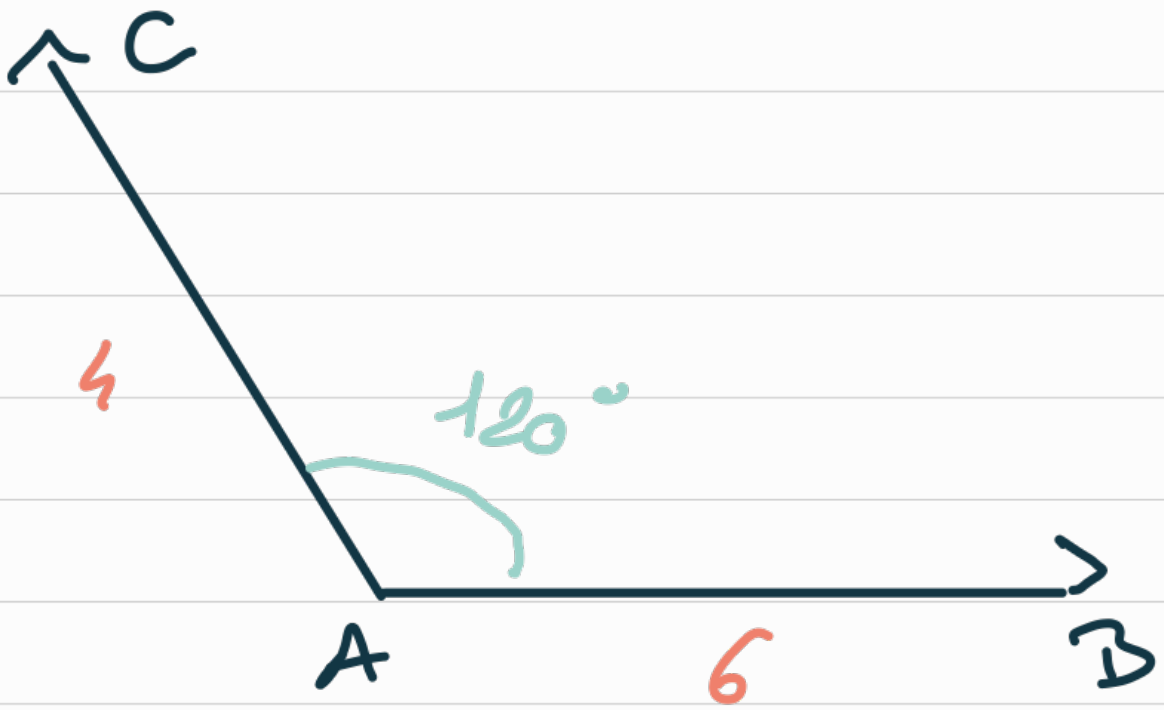
$$= 5.$$

4) $\vec{AB} \cdot \vec{AC} = \frac{1}{2}\left(AB^2 + AC^2 - BC^2\right)$

$$\vec{AB} \cdot \vec{AC} = \frac{1}{2}\left(6^2 + 4^2 - 5^2\right)$$

$$= \frac{27}{2}.$$

5)

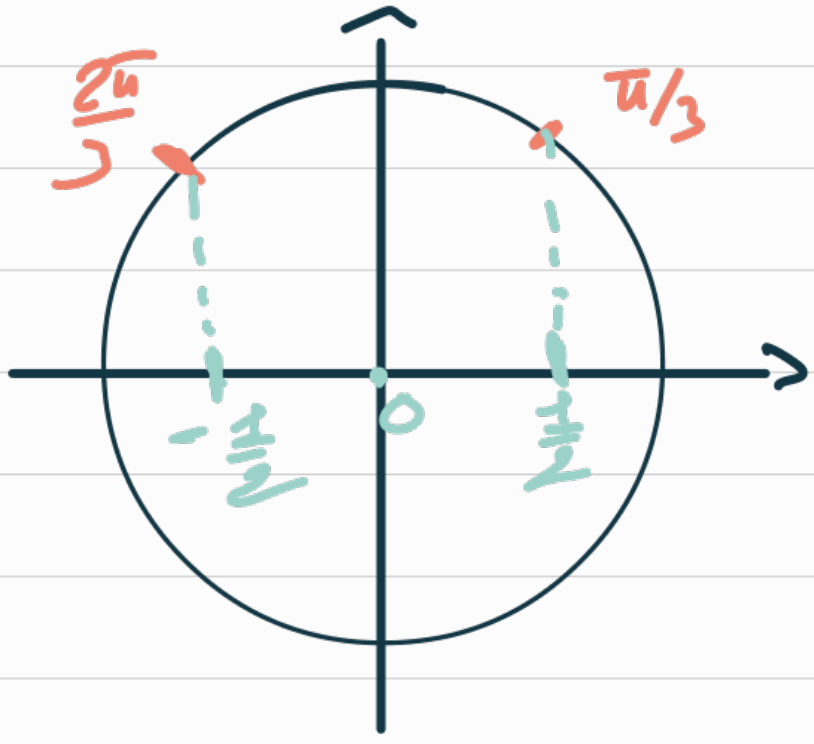


$$\vec{AB} \cdot \vec{AC} = AB \times AC \times \cos\left(\widehat{CAB}\right)$$

$$= 4 \times 6 \times \underbrace{\cos(120°)}_{120° = 2 \times 60°}$$

$$\cos 60° = \frac{1}{2}$$

donc $\cos(120°) = \cos\left(\frac{2\pi}{3}\right)$.



donc $\vec{AB}.\vec{AC} = 4 \times 6 \times -\frac{1}{2} = -12$

6) $\vec{AB}.\vec{AC} = \vec{AB}.\vec{AH}$

$\vec{AB}.\vec{AC} = -AB \times AH$

$-12 \qquad 6$

$\Rightarrow -12 = -6 \times AH$

$\Rightarrow -\frac{12}{-6} = AH \Rightarrow AH = 2$

7) $\vec{AB} \cdot \vec{AC} = AB \times AC \times \cos(\widehat{BAC})$

- $\vec{AB} \cdot \vec{AC} = 4 \times 3 + 3 \times (-1)$

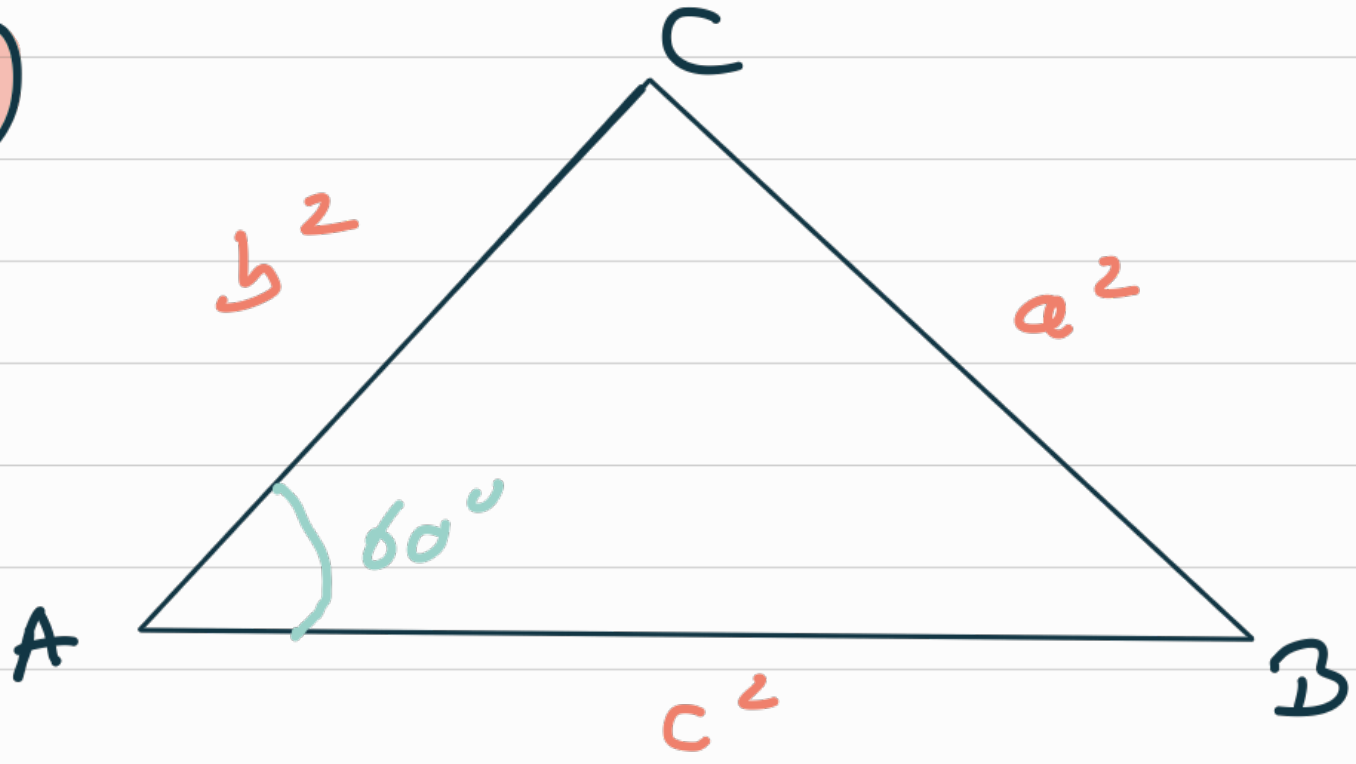
$= 12 - 3 = 9$

- $AB = \sqrt{4^2 + 3^2} = \sqrt{25} = 5$
- $AC = \sqrt{3^2 + (-1)^2} = \sqrt{10}$

d'où $9 = 5 \times \sqrt{10} \times \cos(\widehat{BAC})$

$$\frac{9}{5 \times \sqrt{10}} = \cos(\widehat{BAC}).$$

8)



- $a^2 = b^2 + c^2 - 2bc \times \cos(\widehat{BAC})$

$BC^2 = AC^2 + AB^2 - 2AC \times AB \times \cos(\widehat{BAC})$

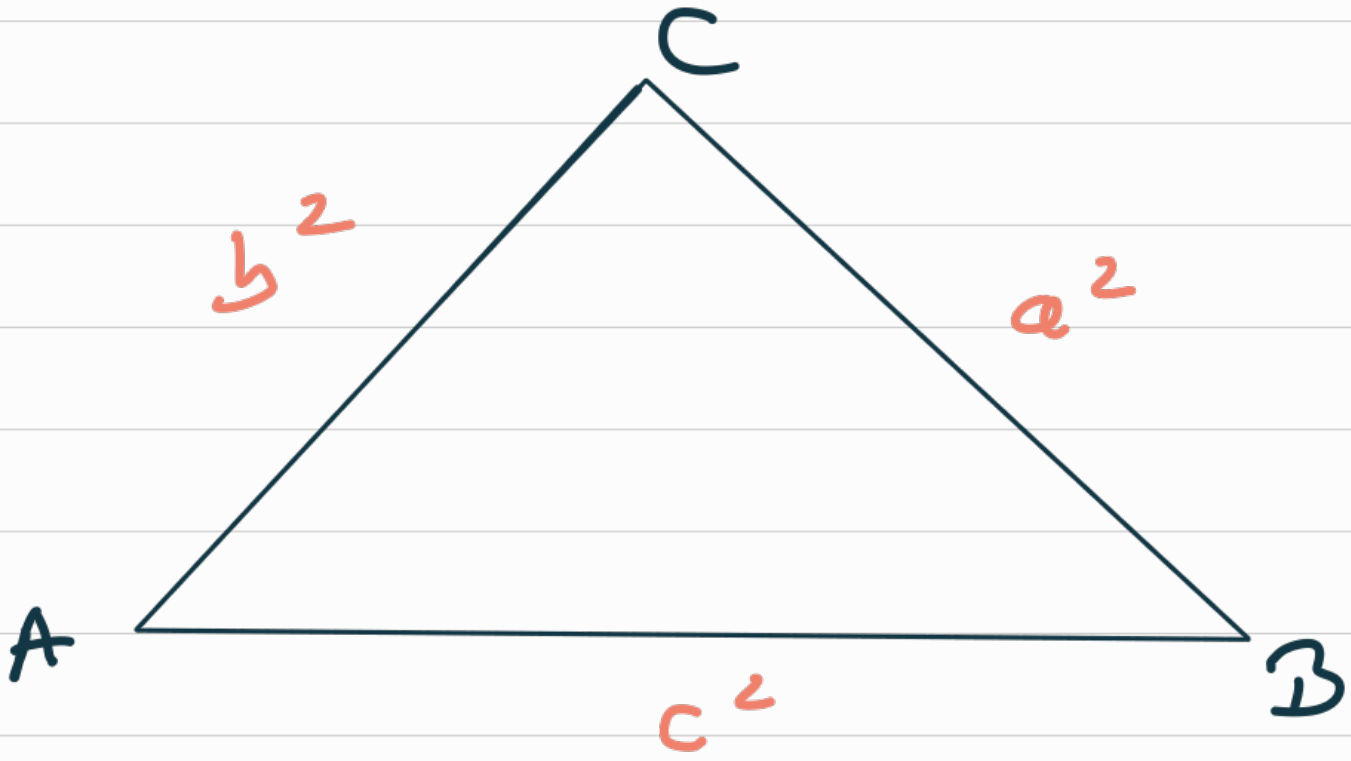
$BC^2 = 2^2 + 4^2 - 2 \times 2 \times 4 \times \cos(60°)$

$BC^2 = 4 + 16 - 16 \times \frac{1}{2}$

$BC^2 = 20 - 8$

$BC^2 = 12 \quad \Rightarrow BC = \sqrt{12}$

exercice 3 :



- $a^2 = b^2 + c^2 - 2bc \times \cos(\widehat{BAC})$

$BC^2 = AC^2 + AB^2 - 2AC \times AB \times \cos(\widehat{A})$

$7^2 = 3^2 + 5^2 - 2 \times 3 \times 5 \times \cos(\widehat{A})$

$7^2 - 3^2 - 5^2 = -2 \times 3 \times 5 \times \cos(\widehat{A})$.

$15 = -30 \times \cos(\widehat{BAC})$

$\frac{15}{-30} = \cos(\widehat{BAC}) \Rightarrow \cos(\widehat{BAC}) = -\frac{1}{2}$