Examen stella

exercice 1 :

a) $(1,1,0,0) \in \text{Vect}\{v_1, v_2\} \cap \text{Vect}\{v_2, v_3, v_4\}$

1. $(1,1,0,0) \in \text{Vect}\{v_1, v_2\}$

$\hookrightarrow (1,1,0,0) = \alpha v_1 + \beta v_2$

2. $(1,1,0,0) \in \text{Vect}\{v_2, v_3, v_4\}$

$\hookrightarrow (1,1,0,0) = \alpha v_2 + \beta v_3 + \gamma v_4$

b) $\underbrace{\mathrm{Vect}\{v_1, v_2\}}_{F} + \underbrace{\mathrm{Vect}\{v_2, v_3, v_4\}}_{G} = \mathbb{R}^4$

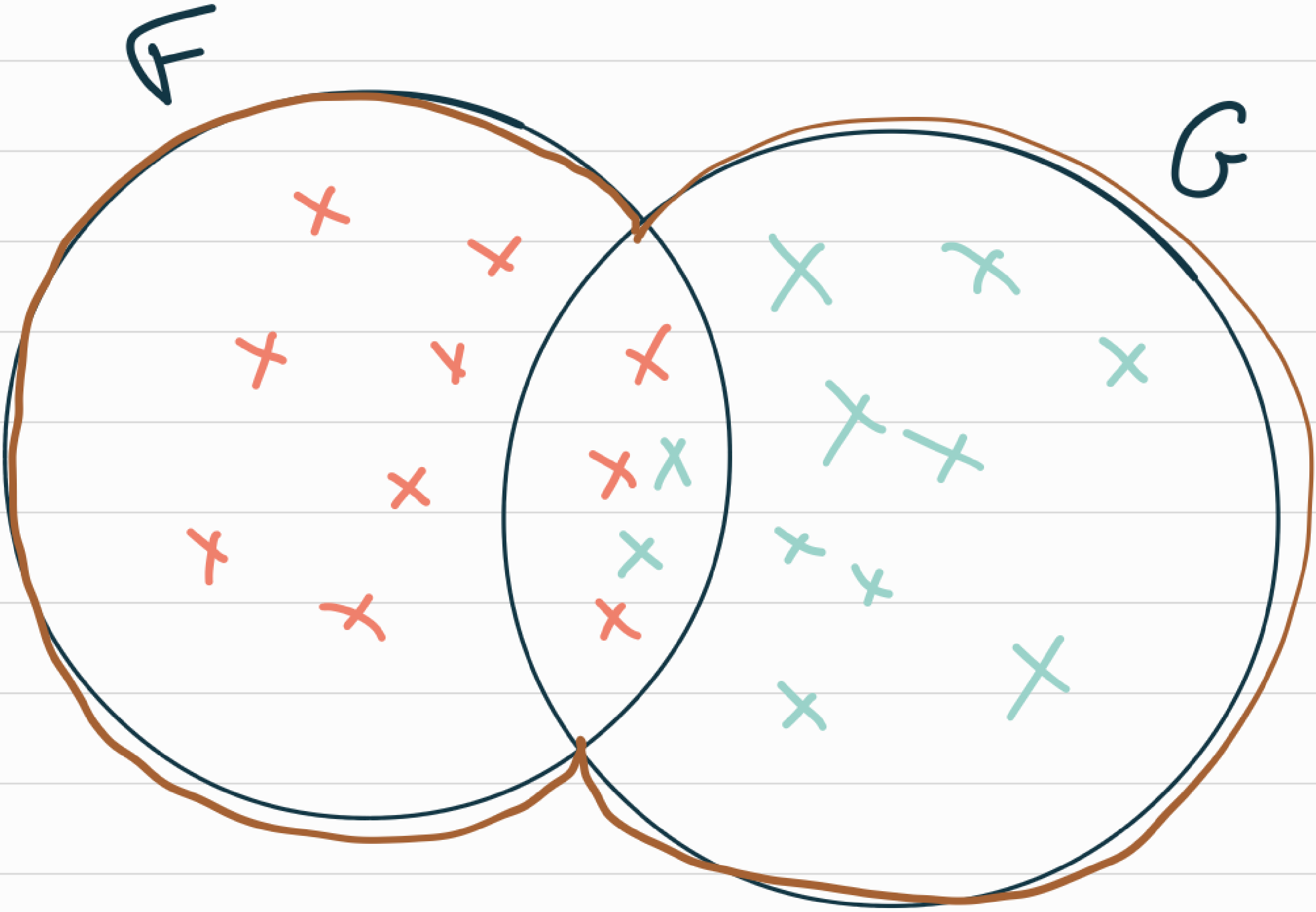
<u>But</u> : $F + G = \mathbb{R}^4$

Soit $(x, y, z, t) \in \mathbb{R}^4$.

$$(x, y, z, t) = \underbrace{f}_{\in F} + \underbrace{g}_{\in G}$$

$$\boxed{\dim(F + G) = \dim(F) + \dim(G) - \dim(F \cap G).}$$

$F = \mathrm{Vect}\{v_1, v_2\} \Rightarrow \dim F = 2$



$G = \text{Vect}\{v_2, v_3, v_4\}$

$\dim(G) = 3$

et $F \cap G = \{v_2\}$

et $\dim(F \cap G) = 1$

donc :

$$\dim(F+G) = \dim(F) + \dim(G) - \dim(F \cap G)$$

$$= 2 + 3 - 1$$

$$= 4 = \dim \mathbb{R}^4.$$

donc $F + G = \mathbb{R}^4$.

exercice :

$$A \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 2 \end{pmatrix}$$

$$P_A(x) = \begin{vmatrix} 1-x & 0 & 0 \\ 0 & -x & -1 \\ 0 & 1 & 2-x \end{vmatrix}$$

$$= (1-x) \begin{vmatrix} -x & -1 \\ 1 & 2-x \end{vmatrix}$$

$$= (1-x)\left[-x(2-x) + 1 \right]$$

$$= (1-x)(x^2 - 2x + 1)$$

$$= (1-x)(x-1)^2 = (1-x)^3$$

donc il y a une $\underbrace{\text{v.p triple}}_{m_a(\lambda_1)=3}$ $\lambda_1 = 1$.

$$E_1 = \ker \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & 1 & 1 \end{pmatrix}$$

$\sim \quad y + z = 0$

$\iff y = -z$ et $x \in \mathbb{R}$.

donc $E_1 = \mathrm{Span}\underbrace{\{(1,0,0), (0,-1,1)\}}_{m_g(\lambda_1)=2}$

Donc A n'est pas diago.

$$\begin{array}{c} \\ J = \begin{pmatrix} \boxed{1} & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \\ \\ \end{array} \begin{array}{l} v_1 \\ v_2 \\ v_3 \end{array}$$

$$Av_3$$

$$P = \begin{array}{c} \begin{pmatrix} 1 & 0 & \\ 0 & -1 & v_3 \\ 0 & 1 & \end{pmatrix} \\ \begin{array}{ccc} v_1 & v_2 & \quad \end{array} \end{array}$$

on pose $v_3(x, y, z)$

et on résout :

$$\boxed{Av_3 = v_2 + v_3}$$

$$A\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} + \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\begin{cases} x = x \\ -z = -1+y \\ y+2z = 1+z \end{cases}$$

$$\Longleftrightarrow \begin{cases} x = x \\ z = 1-y \\ z = 1-y \end{cases}$$

$$\Longleftrightarrow \begin{cases} x = x \\ z = 1-y \end{cases}$$

on pose $v_3(0;1;0)$.

telle que

$$J = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

et $\quad A = PJP^{-1}$.

ou $\quad J = P^{-1}AP$.