Final 2026

exercice 1 :

$$x^2 + y^2 - 2xy - x + 3y = 1.$$

$$\underbrace{Ax^2 + By^2 + Cxy}_{\text{quadratic}} + \underbrace{Dx + Ey}_{\text{linear}} + F = 0$$

1. $$M = \begin{pmatrix} A & \frac{C}{2} \\ \frac{C}{2} & B \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

v.p : $\{2 ; 0\}$

on a $E_2 = \text{Span}\begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$E_0 = \text{Span}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\Rightarrow$ conique est parabole

2 $P = \begin{pmatrix} e_1 & e_2 \end{pmatrix}$

$e_1 = \frac{1}{\sqrt{2}}(1, -1)$

$e_2 = \frac{1}{\sqrt{2}}(1, 1)$

$$P = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix}$$

on fait un changement de variable :

$$\underbrace{\tilde{X}}_{\binom{u}{v}} = P^T X$$

$$= \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$= \begin{pmatrix} \frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} \\ \frac{x}{\sqrt{2}} - \frac{y}{\sqrt{2}} \end{pmatrix}$$

$\cdot\ u = \dfrac{x+y}{\sqrt{2}}$

$\cdot\ v = \dfrac{x-y}{\sqrt{2}}$

$$-x+3y = -\left(\frac{u+v}{\sqrt{2}}\right)+3\left(\frac{u-v}{\sqrt{2}}\right)$$

$$= -\frac{u}{\sqrt{2}} - \frac{v}{\sqrt{2}} + \frac{3}{\sqrt{2}}u - \frac{3v}{\sqrt{2}}$$

$$= \sqrt{2}\,(u-2v)$$

3 x^2+y^2-2xy

$$= \tilde{X}^T D \tilde{X} = (u\ v)\begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix}$$

$= 2v^2$

$$\underbrace{2v^2}_{\text{quadra}} + \underbrace{\sqrt{2}(u-2v)}_{\text{linear}} = 1$$

$$2(v^2 - \sqrt{2}v) + \sqrt{2}u = 1$$

$$2\left[\left(v - \frac{\sqrt{2}}{2}\right)^2 - \frac{1}{2}\right] + \sqrt{2}u = 1$$

$$2\left(v - \frac{\sqrt{2}}{2}\right)^2 - 1 + \sqrt{2}u = 1$$

$$2\left(v - \frac{\sqrt{2}}{2}\right)^2 + \sqrt{2}u = 2$$

donc :

$$\sqrt{2}\, u = 2 - 2\left(v - \frac{\sqrt{2}}{2}\right)^2$$

$$u = \sqrt{2} - \sqrt{2}\left(v - \frac{\sqrt{2}}{2}\right)^2$$

$\Rightarrow$ sommet de $v = \frac{\sqrt{2}}{2}$

et donc $u = \sqrt{2}$

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4) $x = \dfrac{u+v}{\sqrt{2}}$

$y = \dfrac{u-v}{\sqrt{2}}$

donc sommet $(x;y)$

$$x = \frac{\sqrt{2} + \frac{\sqrt{2}}{2}}{2} = \frac{3}{2}$$

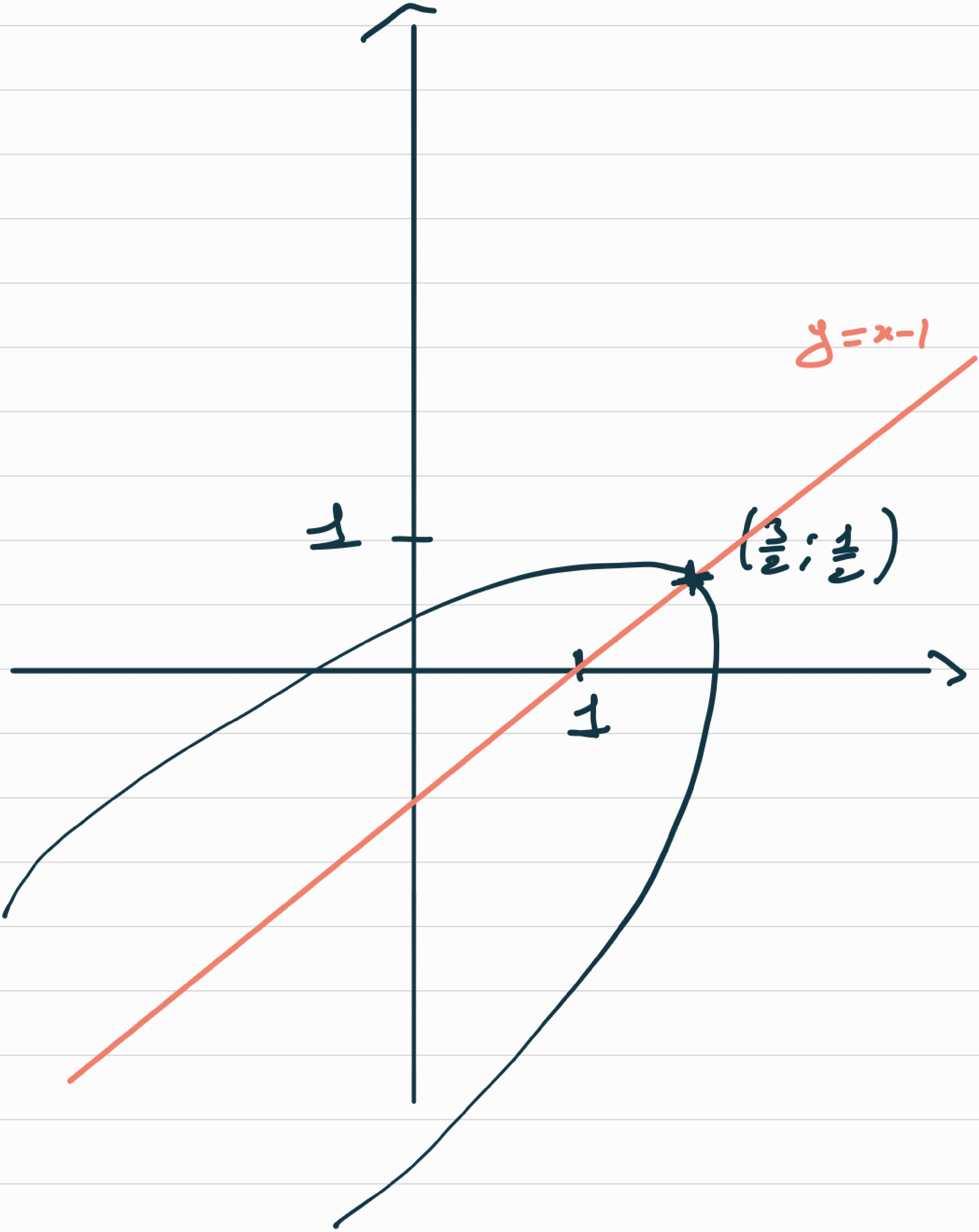
$$y = \frac{\sqrt{2} - \frac{\sqrt{2}}{2}}{2} = \frac{1}{2}$$

$\Rightarrow$ sommet : $\left(\frac{3}{2} ; \frac{1}{2}\right)$

- on regarde le vecteur propre $O : (1;1)$

$\Rightarrow m = \frac{1}{1} = 1$

$\Rightarrow$ la parabole sera le long de la droite $y = 1x - 1$



y=x-1
1
1
(3/2 ; 1/2)