

Final 2026

exercice 1:

$$x^2 + y^2 - 2xy - x + 3y = 1.$$

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$$\underbrace{Ax^2 + By^2 + Cxy}_{\text{quadratic}} + \underbrace{Dx + Ey + F}_{\text{linear}} = 0$$

$$\textcircled{1} \quad M = \begin{pmatrix} A & \frac{C}{2} \\ \frac{C}{2} & B \end{pmatrix}$$
$$= \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

$$v.p : \{2; 0\}$$

$$\text{on a } E_2 = \text{Span} \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$E_0 = \text{Span} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$\Rightarrow$ conique or parabole

$$\textcircled{2} \quad P = \begin{pmatrix} e_1 & e_2 \end{pmatrix}$$

$$e_1 = \frac{1}{\sqrt{2}} (1, -1)$$

$$e_2 = \frac{1}{\sqrt{2}} (1, 1)$$

$$P = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix}$$

on fait un changement de variable :

$$\begin{aligned} \begin{pmatrix} x \\ y \end{pmatrix} &= P^T X \\ &= \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix} \\ &= \begin{pmatrix} \frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} \\ \frac{x}{\sqrt{2}} - \frac{y}{\sqrt{2}} \end{pmatrix} \end{aligned}$$

$$, u = \frac{x+y}{\sqrt{2}}$$

$$, v = \frac{x-y}{\sqrt{2}}$$

$$-x + 3y = -\left(\frac{u+v}{\sqrt{2}}\right) + 3\left(\frac{u-v}{\sqrt{2}}\right)$$

$$= -\frac{u}{\sqrt{2}} - \frac{v}{\sqrt{2}} + \frac{3}{\sqrt{2}}u - \frac{3v}{\sqrt{2}}$$

$$= \sqrt{2}(u - 2v)$$

$$\textcircled{3} \quad x^2 + y^2 - 2xy$$

$$= \tilde{X}^T D \tilde{X} = (u \ v) \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix}$$

$$= 2v^2$$

$$\underbrace{2v^2}_{\text{quadr}} + \underbrace{\sqrt{2}(u-2v)}_{\text{linear}} = 1$$

$$2(v^2 - \sqrt{2}v) + \sqrt{2}u = 1$$

$$2\left[\left(v - \frac{\sqrt{2}}{2}\right)^2 - \frac{1}{2}\right] + \sqrt{2}u = 1$$

$$2\left(v - \frac{\sqrt{2}}{2}\right)^2 - 1 + \sqrt{2}u = 1$$

$$2\left(v - \frac{\sqrt{2}}{2}\right)^2 + \sqrt{2}u = 2$$

done :

$$\sqrt{2} u = 2 - 2 \left(v - \frac{\sqrt{2}}{2} \right)^2$$

$$u = \sqrt{2} - \sqrt{2} \left(v - \frac{\sqrt{2}}{2} \right)^2$$

$$\Rightarrow \text{Sommet de } v = \frac{\sqrt{2}}{2}$$

$$\text{et donc } u = \sqrt{2}$$

$$\textcircled{4} \quad x = \frac{u+v}{\sqrt{2}}$$

$$y = \frac{u-v}{\sqrt{2}}$$

donc sommet $(x; y)$

$$x = \frac{\sqrt{2} + \frac{\sqrt{2}}{2}}{2} = \frac{3}{2}$$

$$y = \frac{\sqrt{2} - \frac{\sqrt{2}}{2}}{2} = \frac{1}{2}$$

$\Rightarrow$ sommet : $\left(\frac{3}{2}; \frac{1}{2}\right)$

• on regarde le vecteur propre $O : (1; 1)$

$\Rightarrow$ la parabole sera de la droite $y = x - 1$

(Note: In the original image, there are red annotations. A red arrow points from the vector $(1; 1)$ to the slope $m = \frac{1}{1} = 1$. Another red arrow points from the slope 1 to the coefficient of x in the equation $y = x - 1$.)

