Dérivation

- La fonction dérivée de la fonction f est f'

- $f' \geq 0 \Rightarrow f$ est croissante
- $f' \leq 0 \Rightarrow f$ est décroissante

- Formules à apprendre :

$$(ax+b)' = a$$

$$(3x+2)' = 3$$

$$(x^2)' = 2x$$

$$(x^3)' = 3x^2$$

$$(x^n)' = n x^{n-1}$$

$$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$$

$$\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$$

$$\left(\frac{1}{x^n}\right)' = -\frac{n}{x^{n+1}}$$

exercice :

- $(-5x+100)' = -5$
- $(x^3 + x^2)' = 3x^2 + 2x$
- $(3\sqrt{x})' = 3 \times \frac{1}{2\sqrt{x}} = \frac{3}{2\sqrt{x}}$
- $\left(\frac{1}{x^5}\right)' = \frac{-5}{x^6}$

$$\left(u(x)+v(x)\right)' = u'(x)+v'(x)$$

$$\left(k\,u(x)\right)' = k \times u'(x)$$

$$\left(u(x) \times v(x)\right)' = u'(x)v(x) + u(x)v'(x)$$

$$\left(\frac{u(x)}{v(x)}\right)' = \frac{u'(x)v(x) - u(x)v'(x)}{\left(v(x)\right)^2}$$

exercices :

- $(3x^2 + 4\sqrt{x})' = 3 \times 2x + 4 \times \frac{1}{2\sqrt{x}}$

$$= 6x + \frac{2}{\sqrt{x}}$$

- $(5x^3 - 3x^2)'$

$$= 5 \times 3x^2 - 6x$$

$$= 15x^2 - 6x$$

- $f(x) = \underbrace{(3x^2+4x)}_{u(x)}\underbrace{(5x-1)}_{v(x)}$

$$\begin{cases} u(x) = 3x^2+4x \\ u'(x) = 6x+4 \\ v(x) = 5x-1 \\ v'(x) = 5 \end{cases}$$

$$f'(x) = \overset{u'}{(6x+4)}\overset{v}{(5x-1)} + \overset{u}{(3x^2+4x)} \times \overset{v'}{5}$$

$$= 30x^2 - 6x + 20x - 4 + 15x^2 + 20x$$

$$= 45x^2 + 34x - 4$$

$$\cdot \quad f(x) = \frac{\overbrace{6x - 5}^{u}}{\underbrace{x^2 - 2x - 1}_{v}}$$

$$u(x) = 6x - 5$$

$$u'(x) = 6$$

$$v(x) = x^2 - 2x - 1$$

$$v'(x) = 2x - 2$$

$$f'(x) = \frac{6(x^2 - 2x - 1) - (6x-5)(2x-2)}{(x^2 - 2x - 1)^2}$$

$$= \frac{6x^2 - 12x - 6 - (12x^2 - 12x - 10x + 10)}{(x^2 - 2x - 1)^2}$$

$$= \frac{6x^2 - 12x - 6 - 12x^2 + 12x + 10x - 10}{(x^2 - 2x - 1)^2}$$

$$= \frac{-6x^2 + 10x - 16}{(x^2 - 2x - 1)^2}$$

exercice :

- $f(x) = (3x^2+4x)(5x-1)$

1) Calculer la dérivée

$f'(x) = 45x^2 + 34x - 4$

2) Etudier le signe de f'

$\Delta = b^2 - 4ac$

$= 34^2 - 4 \times 45 \times (-4)$

$= 1876 > 0$

$$x_1 = \frac{-b-\sqrt{\Delta}}{2a} \simeq -0,86$$

$$x_2 = \frac{-b+\sqrt{\Delta}}{2a} \simeq 0,10$$

x	$-\infty$		$-0,86$		$0,10$		$+\infty$
signe f'		$+$	0	$-$	0	$+$	

3) Dresser le tableau de variation

x	$-\infty$		$-0{,}86$		$0{,}10$		$+\infty$
f'		$+$	0	$-$	0	$+$	
f		$\nearrow$		$\searrow$		$\nearrow$	

$$f(x) = \frac{6x-5}{x^2-2x-1}$$

$$f'(x) = \frac{-6x^2+10x-16}{(x^2-2x-1)^2}$$

$(x^2-2x-1)^2 > 0$

$\hookrightarrow$ v.p $= \begin{cases} -0{,}41 \\ 2{,}41 \end{cases}$

$$\Delta = b^2 - 4ac$$

$$= 100 - 4 \times (-6) \times (-16)$$

$$= -284 < 0$$

x	$-\infty$ $-0{,}41$ $2{,}41$ $+\infty$
$(x^2-2x-1)^2$	$+$
$-6x^2+10x-16$	$-$

x	$-\infty$		$-0{,}41$		$2{,}41$		$+\infty$
f'		$-$	$\Vert$	$-$	$\Vert$	$-$	
f		$\searrow$	$\Vert$	$\searrow$	$\Vert$	$\searrow$	