

exercice 2:

$$(p \Rightarrow q) \wedge (q \Rightarrow r)$$

a

$$\Rightarrow p \Rightarrow r$$

p	q	r	$p \Rightarrow q$	$q \Rightarrow r$	$(p \Rightarrow q) \wedge (q \Rightarrow r)$	$p \Rightarrow r$
1	1	1	1	1	1	1
1	1	0	1	0	0	0
1	0	0	0	1	0	0
1	0	1	0	1	0	0
0	1	1	1	1	1	1
0	1	0	1	0	0	1

[illegible]

exercice 6 :

$$u_n = \left(1 + \frac{1}{n}\right)^n, \quad n \geq 1$$

a) Soit $n \geq 1$.

$$(a+b)^n = \sum_{k=0}^n \binom{n}{k} a^k b^{n-k}$$

$$u_n = \sum_{k=0}^n \binom{n}{k} 1^{n-k} \left(\frac{1}{n}\right)^k$$

$$= \sum_{k=0}^n \binom{n}{k} \frac{1}{n^k}$$

$$= 1 + \binom{n}{1} \frac{1}{n} + \sum_{k=2}^n \binom{n}{k} \frac{1}{n^k}$$

$$\geq 1 + n \times \frac{1}{n} = 2$$

Exercise 5:

$$a_n = \frac{2^n \cos(n) + 25^{n+1}}{3^n \sin(n) + 5^{2n-1}}$$

$$= \frac{2^n \cos(n) + 25^n \times 25}{3^n \sin(n) + (5^2)^n \times 5^{-1}}$$

$$= \frac{2^n \cos(n) + 25^n \times 25}{3^n \sin(n) + 25^n \times 5^{-1}}$$

$$= \frac{\cancel{25^n} \left[\frac{2^n \cos(n)}{\cancel{25^n}} + \frac{\cancel{25^n} \times 25}{\cancel{25^n}} \right]}{\cancel{25^n} \left[\frac{3^n \sin(n)}{\cancel{25^n}} + \frac{\cancel{25^n} \times \frac{1}{5}}{\cancel{25^n}} \right]}$$

$$\text{car } -1 \leq \cos(n) \leq 1$$

$$\underbrace{0 \leftarrow}_{\text{}} -\left(\frac{2}{25}\right)^n \leq \left(\frac{2}{25}\right)^n \cos(n) \leq \left(\frac{2}{25}\right)^n \rightarrow \underbrace{0}_{\text{}}$$

$$\text{car } \lim_{n \rightarrow +\infty} \left(\frac{2}{25}\right)^n = 0$$

$$\text{car } -1 < \frac{2}{25} < 1$$

donc par encadrement on a

$$\lim_{n \rightarrow +\infty} \left(\frac{2}{25}\right)^n \cos(n) = 0$$

$$\text{De même } \lim_{n \rightarrow +\infty} \left(\frac{2}{25}\right)^n \sin(n) = 0.$$

$$\text{Donc : } a_n = \frac{\left(\frac{2}{25}\right)^n \cos(n) + 25}{\left(\frac{2}{25}\right)^n \sin(n) + \frac{1}{5}}$$

$$\lim_{n \rightarrow \infty} a_n = \frac{0 + 25}{0 + \underline{\underline{1}}}$$

$$= \frac{25}{\underline{\underline{1}}}$$

$$= 25 \times \frac{5}{1}$$

$$= 125.$$

exercice 7 :

$\forall$: pour tout

$\exists$: il existe

$\exists!$: il existe un unique

$\sim$: négation

a)

$\forall n \in \mathbb{R}, \exists D \in \mathbb{R},$

$\forall x < D \Rightarrow f(x) < n.$

$\hookrightarrow \lim_{x \rightarrow -\infty} f(x) = -\infty$

$\lim_{x \rightarrow +\infty} f(x) = +\infty$

3) $\sim A$:

$\exists n \in \mathbb{R}, \forall D \in \mathbb{R},$

$\exists x < D$ tal que $f(x) > n$

exercice 8 :

c) $\lim_{x \rightarrow 0^+}$ $x^{\frac{1}{1+\ln(x)}}$

$$x^a = \exp(a \ln(x))$$

$$x^{\frac{1}{1+\ln(x)}}$$

$$= \exp\left(\frac{1}{1+\ln(x)} \times \ln(x)\right)$$

$$= \exp\left(\frac{\ln(x)}{1+\ln(x)}\right)$$

$$\text{or } \frac{\ln(x)}{1+\ln(x)} = \frac{\cancel{\ln(x)} \times 1}{\cancel{\ln(x)} \left(\frac{1}{\ln(x)} + 1\right)}$$

$$= \frac{1}{\frac{1}{h(x)} + 1}$$

or $\lim_{x \rightarrow 0^+} h(x) = -\infty,$

hence $\lim_{x \rightarrow 0^+} \frac{1}{h(x)} = 0.$

Hence $\lim_{x \rightarrow 0^-} \frac{1}{h(x)} + 1 = 1$

Ainsi: $\lim_{x \rightarrow 0^+} \frac{1}{\frac{1}{h(x)} + 1} = \frac{1}{1} = 1$

hence :

$$\lim_{x \rightarrow 0^+} x^{\frac{1}{1+h(x)}} = e^1 = e$$