

Limites de fonctions

exercice 14:

a) $\lim_{x \rightarrow +\infty} (3\sqrt{x} - 2) ?$

$$\lim_{x \rightarrow +\infty} \sqrt{x} = +\infty, \text{ et donc}$$

$$\lim_{x \rightarrow +\infty} 3\sqrt{x} = +\infty, \text{ par somme}$$

$$\lim_{x \rightarrow +\infty} 3\sqrt{x} - 2 = +\infty.$$

b) $\lim_{x \rightarrow -\infty} (-8x + 4x^2).$

$$\lim_{x \rightarrow -\infty} -8x = +\infty$$

$$\lim_{x \rightarrow -\infty} 4x^2 = +\infty.$$

donc par somme de limites

$$\lim_{x \rightarrow -\infty} -8x + 4x^2 = +\infty.$$

Déterminer les limites suivantes

$$\lim_{x \rightarrow +\infty} (x^3 + x - 3)$$

$$\lim_{x \rightarrow +\infty} (x^3 + x^2 - 3) = +\infty$$

$$\lim_{x \rightarrow +\infty} \left(\frac{1}{1 + e^{-x}} \right)$$

$$\lim_{x \rightarrow +\infty} \left(\frac{1}{x^3} + 4\sqrt{x} \right) = +\infty$$

$$\lim_{x \rightarrow -\infty} (e^{-x^2+7x-3})$$

$$\lim_{x \rightarrow -\infty} (x^3 + x - 3) = -\infty$$

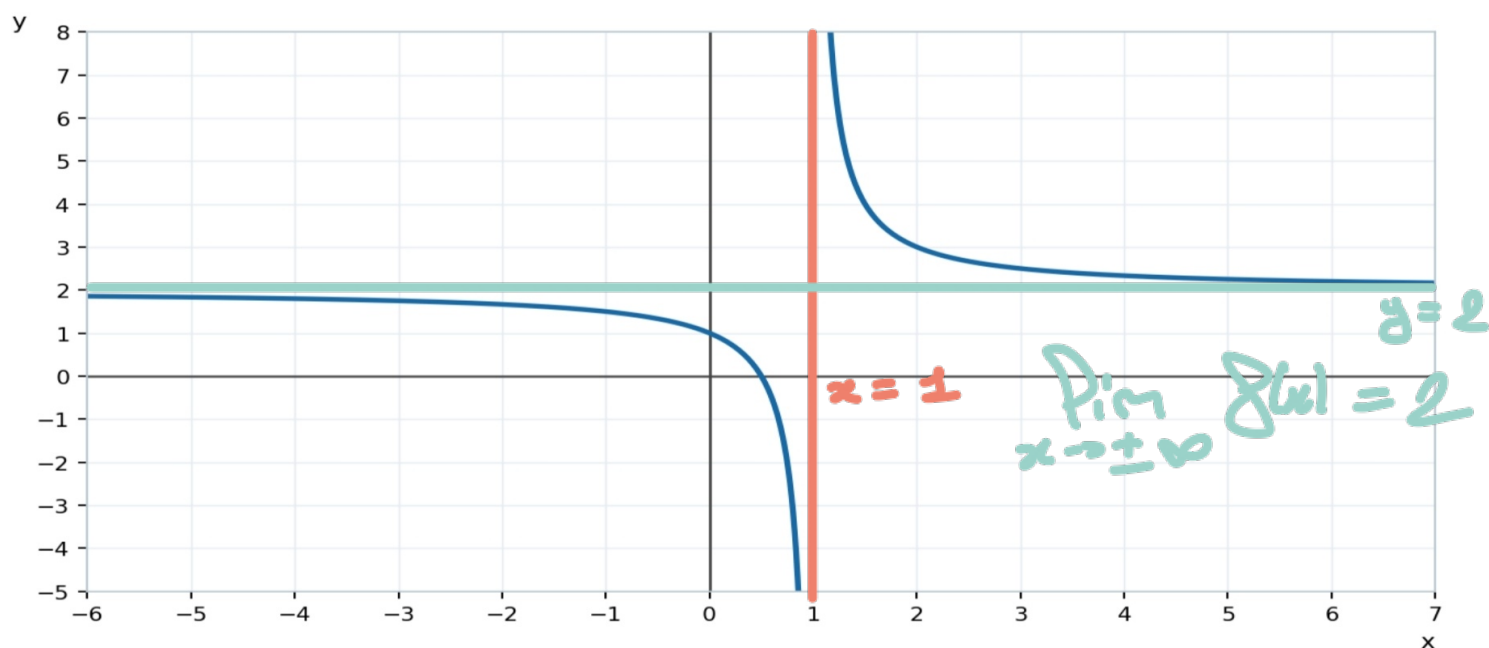
$$\lim_{x \rightarrow -\infty} (-2x^3 + x^2 - 3)$$

$$\lim_{x \rightarrow -\infty} \left(\frac{1}{1 + e^{-x}} \right) = 0$$

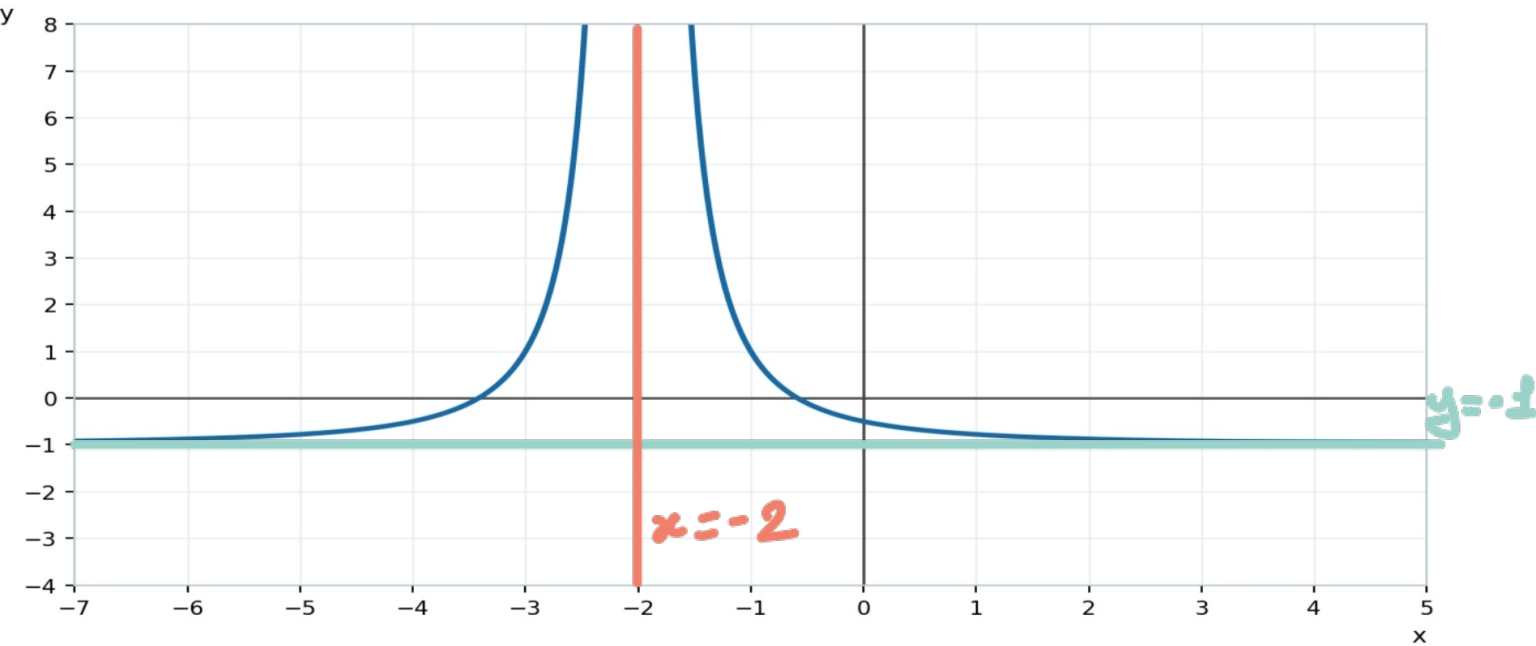
$$\lim_{x \rightarrow 0^+} \left(\frac{1}{x^3} + 4\sqrt{x} \right)$$

$$\lim_{x \rightarrow +\infty} ((1 - 2x)e^x)$$

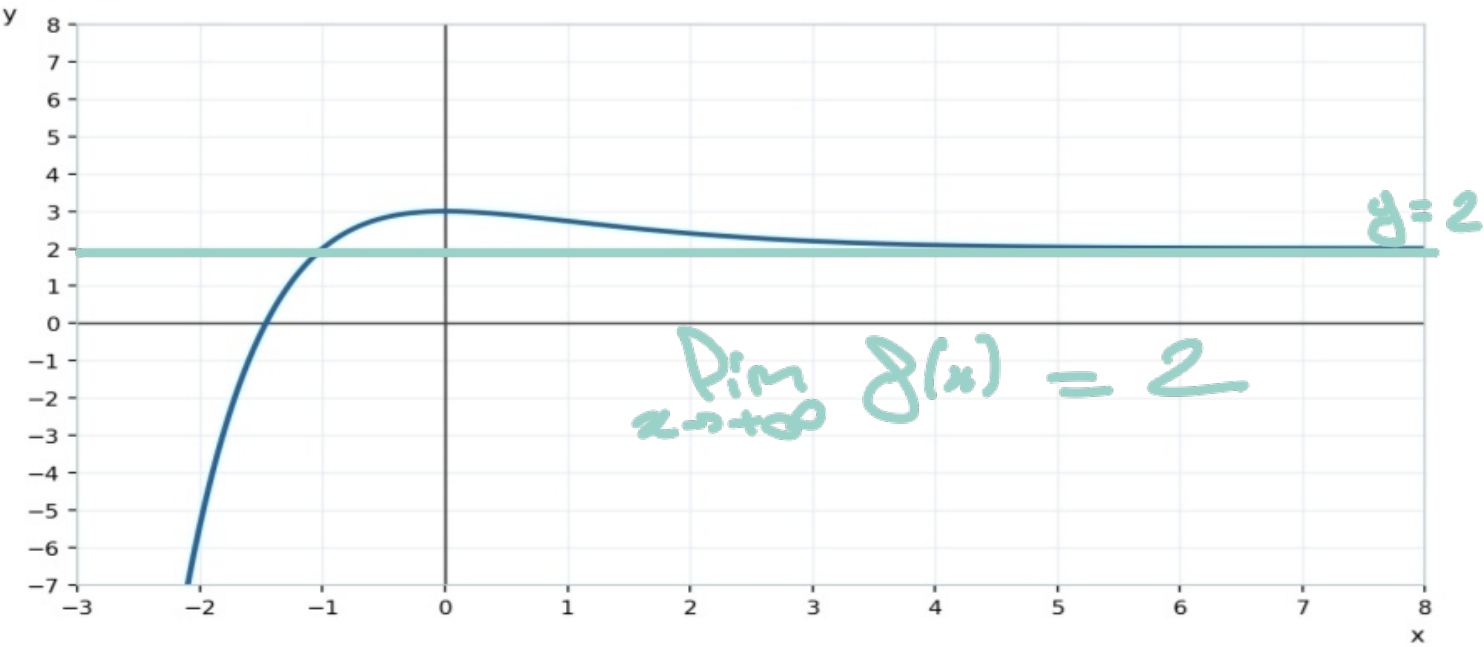
Graphique 1



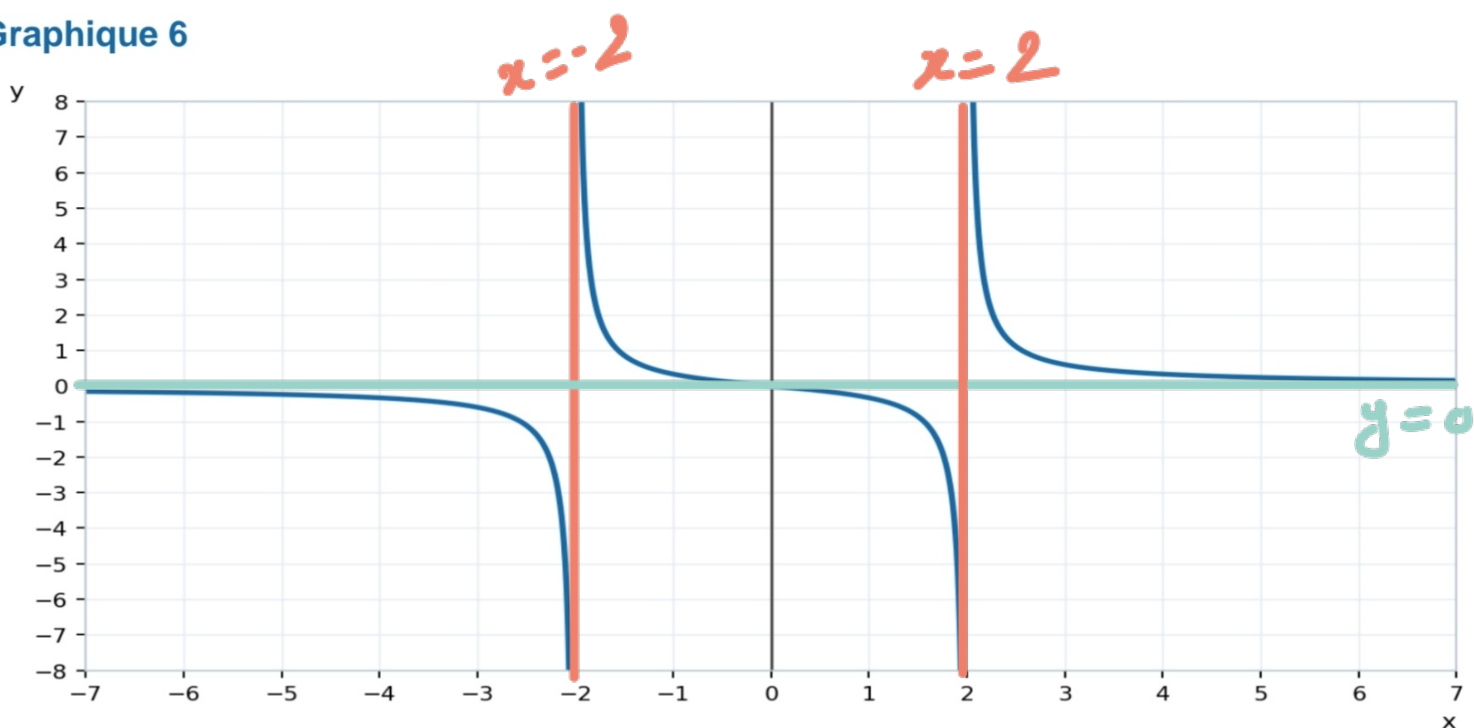
Graphique 2



Graphique 5



Graphique 6



exercice 144 :

$$-1 \leq \sin(x) \leq 1$$

$$-5 \leq 5\sin(x) \leq 5$$

$$-5-2x \leq 5\sin(x) - 2x \leq 5-2x$$

$$\frac{-5-2x}{x-3} \leq \frac{5\sin(x) - 2x}{x-3} \leq \frac{5-2x}{x-3}$$

$$\text{or } \frac{5-2x}{x-3} = \frac{\cancel{x} \left(\frac{5}{\cancel{x}} - 2 \right)}{\cancel{x} \left(1 - \frac{3}{\cancel{x}} \right)}$$

$$= \frac{\frac{5}{x} - 2}{1 - \frac{3}{x}}$$

$$\text{or } \lim_{x \rightarrow +\infty} \frac{5}{x} = \lim_{x \rightarrow +\infty} \frac{3}{x} = 0$$

$$\text{donc par somme } \lim_{x \rightarrow +\infty} \frac{5}{x} - 2 = -2$$

$$\text{et } \lim_{x \rightarrow +\infty} 1 - \frac{3}{x} = 1.$$

$$\text{Par quotient, } \lim_{x \rightarrow +\infty} \frac{\frac{5}{x} - 2}{1 - \frac{3}{x}} = -2$$

$$\text{donc } \lim_{x \rightarrow +\infty} \frac{5 - 2x}{x - 3} = -2$$

$$\text{De même, } \lim_{x \rightarrow +\infty} \frac{-5 - 2x}{x - 3} = -2$$

$$\text{Par le théorème d'encadrement, } \lim_{x \rightarrow +\infty} h(x) = -2.$$

$$\lim_{\substack{x \rightarrow 3 \\ x < 3}} h(x) = ?$$

$$\lim_{x \rightarrow 3^-} 5\sin(x) - 2x$$

$$= 5\sin(3) - 2 \times 3$$

$$= 5\sin(3) - 6$$

$$\approx -5,7 < 0$$

$$\lim_{\substack{x \rightarrow 3 \\ x < 3}} \frac{1}{x-3} = -\infty$$

$$\frac{1}{0^-} = -\infty$$

$$\frac{1}{0^+} = +\infty$$

Ainsi par produit,

$$\lim_{x \rightarrow 3^-} h(x) = -\infty.$$

$$\lim_{\substack{x \rightarrow 2 \\ x > 2}} \frac{x^3 - 4}{x - 2} = ?$$

$x - 2 > 0$

$$\begin{array}{r|l} x & 2 \rightarrow \\ \hline x^2 & - \phi + \end{array}$$

$$\lim_{\substack{x \rightarrow 2 \\ x > 2}} \frac{1}{x - 2} = +\infty$$

0^+

et $\lim_{\substack{x \rightarrow 2 \\ x > 2}} 2^3 - 4 = 8 - 4 = 4 > 0$

donc $\lim_{\substack{x \rightarrow 2 \\ x > 2}} \frac{x^3 - 4}{x - 2} = +\infty$

$$\lim_{\substack{x \rightarrow 3 \\ x < 3}} \frac{2 > 0}{x - 3} = -\infty$$

$\hookrightarrow x - 3 < 0$

$$x < 3 \quad (\Rightarrow) \quad x - 3 < 0$$

clon

$$\lim_{x \rightarrow 3^-} \frac{1}{\underbrace{x-3}_0^-} = -\infty$$

$$\lim_{\substack{x \rightarrow 4 \\ x > 4}} \frac{2x - 3}{x - 4} = +\infty.$$

$\swarrow \quad \begin{matrix} 2x-3 \\ = 5 > 0 \end{matrix}$

$\hookrightarrow 0^+$