

Analyse πD3

exercice 3:

$$3) \cot(x + T) = \frac{\cos(x + T)}{\sin(x + T)}$$

$$\text{or } \cos(x + 2\pi) = \cos(x)$$

$$\sin(x + 2\pi) = \sin(x).$$

$$\text{on pose } T = 2\pi$$

$$\text{after } \cot(x + 2\pi) = \frac{\cos(x + 2\pi)}{\sin(x + 2\pi)}$$

$$= \frac{\cos(x)}{\sin(x)}$$

$$= \cot(x)$$

donc 2π est une période.

$$\text{or } \cot(x + \pi)$$

$$= \frac{\cos(x + \pi)}{\sin(x + \pi)}$$

$$= \frac{-\cos(x)}{-\sin(x)}$$

$$= \frac{\cos(x)}{\sin(x)}$$

$$= \cot(x)$$

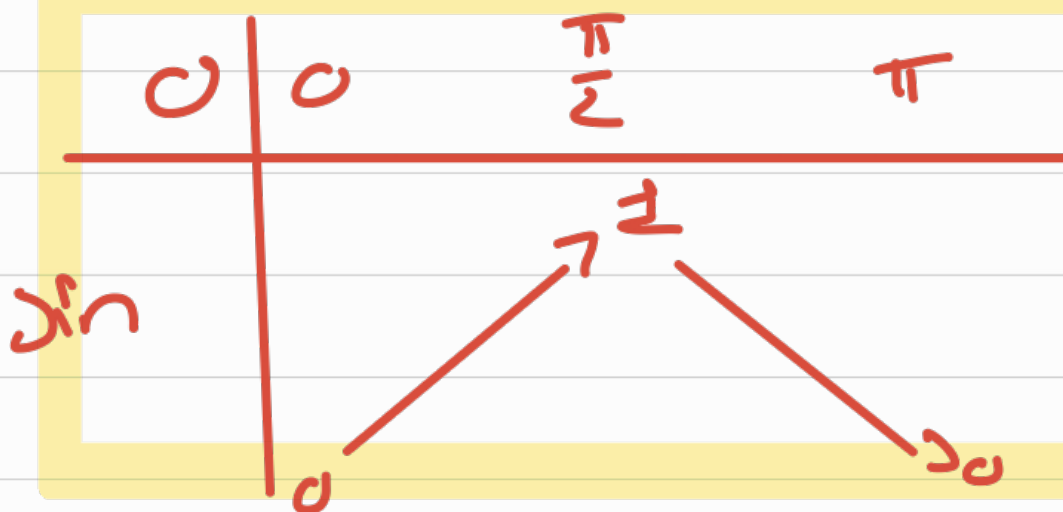
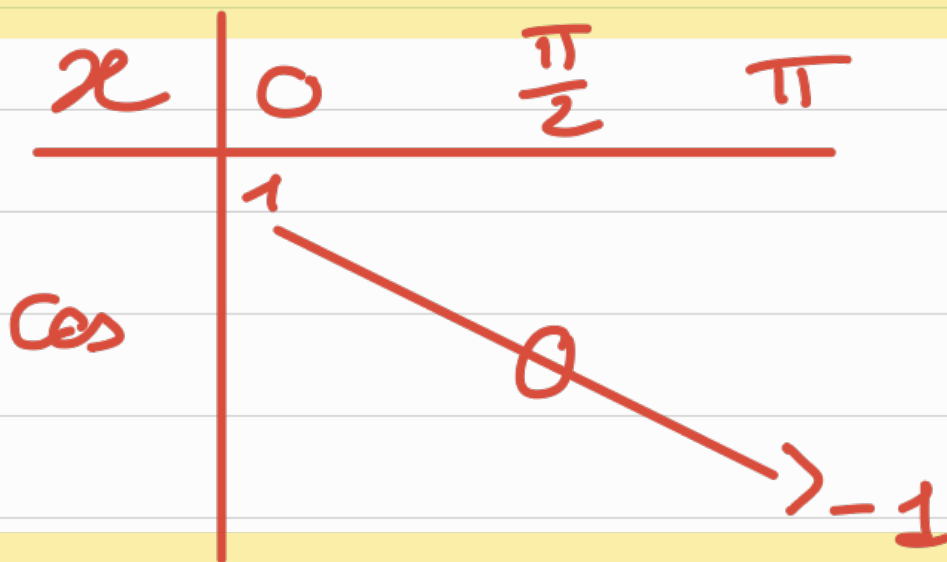
donc $\cot$ est π -périodique.

exercice 2 :

$$\cos'(x) = -\sin(x)$$

$$\sin(x) = \cos(x)$$

3) $J = [0; \frac{\pi}{3}]$

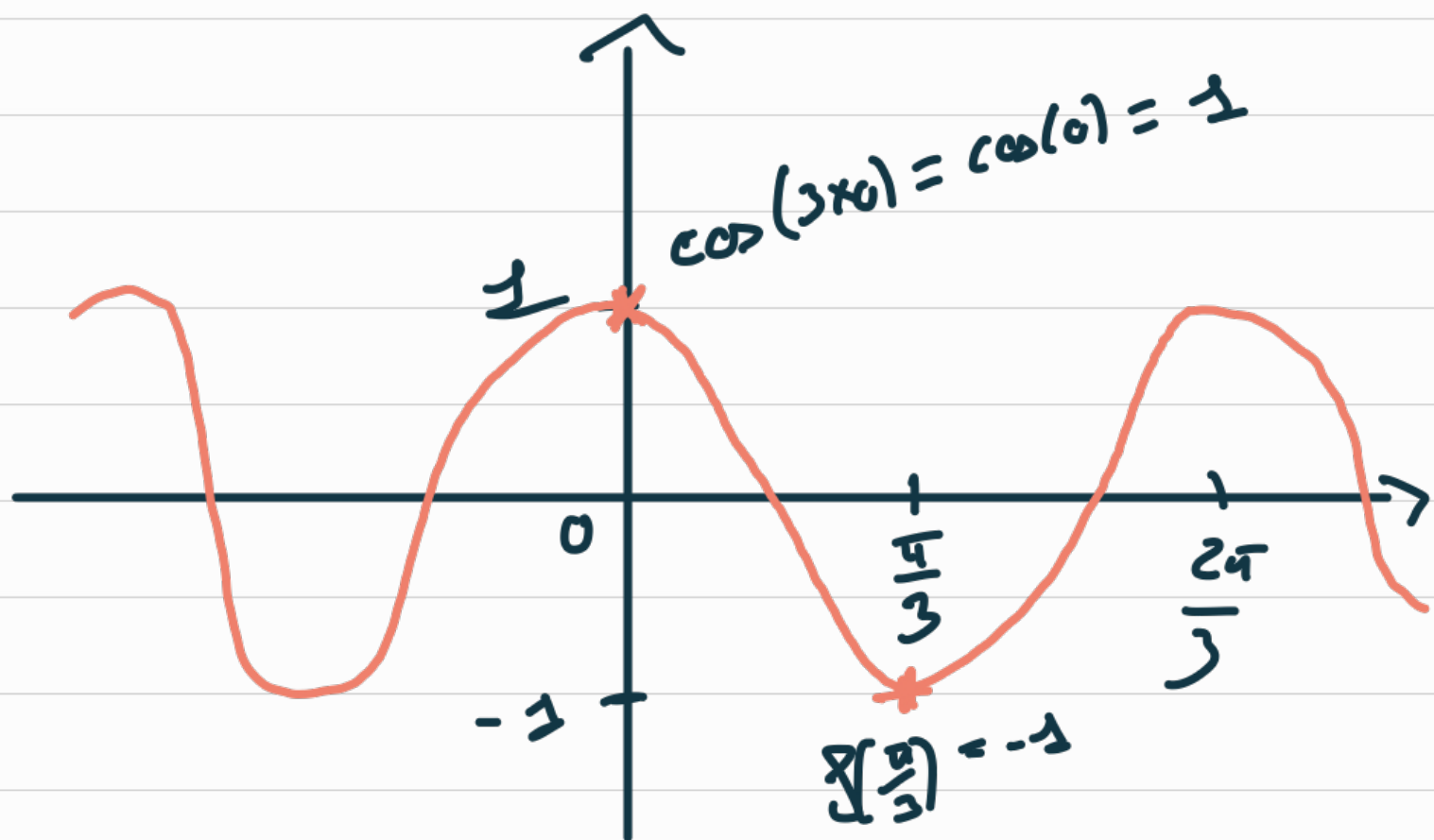


or sur $[0, \pi]$ cos est
décroissante.

$$\text{Si } 0 \leq x \leq \frac{\pi}{3} \quad \times 3 \downarrow \quad \uparrow \times 3$$
$$0 \leq 3x \leq \pi$$

donc comme $x \mapsto 3x$ est
croissante sur $\mathbb{R}$

Par composition, f est
strictement décroissante sur J .



exercice 5 :

$$2) \ln\left(\frac{1}{3}\right) + \ln\left(\frac{3}{4}\right) + \ln\left(\frac{4}{7}\right) + \ln\left(\frac{7}{8}\right)$$

$$= \ln\left(\frac{1}{\cancel{3}} \times \cancel{3} \times \cancel{4} \times \cancel{7} \times \frac{7}{8}\right)$$

$$= \ln\left(\frac{1}{8}\right)$$

$$= -\ln(8)$$

$$\ln\left(\frac{1}{a}\right) = -\ln(a)$$

$$1) \ln(\sqrt{e}) = \frac{1}{2} \ln(e)$$

$$= \frac{1}{2} \times 1 = \frac{1}{2}$$

exercice 6 :

$$2) \ln(x^2 - 1) = \ln(4) + \ln(2)$$

$$\Leftrightarrow \ln(x^2 - 1) = \ln(8)$$

$$\Leftrightarrow x^2 - 1 = 8$$

$$\Leftrightarrow x^2 = 9$$

$$\Leftrightarrow x = 3 \quad \text{ou} \quad x = -3.$$

$$S = \{3; -3\}$$

$$1) \ln(x^3) = -3$$

$$\Leftrightarrow x^3 = e^{-3}$$

$$\Leftrightarrow x = (e^{-3})^{\frac{1}{3}}$$

$$\Leftrightarrow x = e^{-3 \times \frac{1}{3}} = e^{-1} = \frac{1}{e}$$

$$e^a \times e^b = e^{a+b}$$

$$\frac{e^a}{e^b} = e^{a-b}$$

$$e^{-a} = \frac{1}{e^a}$$

$$(e^a)^n = e^{a \times n}$$

$$\ln(a \times b) = \ln(a) + \ln(b)$$

$$\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$$

$$\ln\left(\frac{1}{a}\right) = -\ln(a)$$

$$\ln(a^n) = n \ln(a).$$

$$4) e^{3x+4} = 2$$

$$\Leftrightarrow 3x+4 = \ln(2)$$

$$\Leftrightarrow 3x = \ln(2) - 4$$

$$\Leftrightarrow x = \frac{\ln(2) - 4}{3}$$

$$\Rightarrow \left\{ \frac{\ln(2) - 4}{3} \right\}$$

exercice 7:

$$1) 3 - \ln(x) \leq 0$$

$$\Leftrightarrow 3 \leq \ln(x)$$

$$\Leftrightarrow e^3 \leq x$$

$$\text{donc } \mathcal{S} = [e^3; +\infty[$$

$$2) \quad 1 - 2h(2x) > 0$$

$$\Leftrightarrow 1 > 2h(2x)$$

$$\Leftrightarrow \frac{1}{2} > h(2x)$$

$$\Leftrightarrow e^{1/2} > 2x$$

$$\Leftrightarrow \frac{e^{1/2}}{2} > x$$

$$\text{donc }] =] - \infty; \frac{e^{1/2}}{2}] .$$

exercice 8:

1) Soit $x \in \mathbb{R}$.

$$\frac{1 - e^{-2x}}{1 + e^{-2x}} \Rightarrow \frac{e^{2x} - 1}{e^{2x} + 1}$$

$$\Leftrightarrow (1 - e^{-2x})(e^{2x} + 1) = (1 + e^{-2x})(e^{2x} - 1)$$

$$\Leftrightarrow (1 - e^{-2x})(e^{2x} + 1)$$

$$= e^{2x} + 1 - e^{-2x} \times e^{2x} - e^{-2x}$$

$$= e^{2x} + 1 - 1 - e^{-2x}$$

$$= e^{2x} - e^{-2x}$$

$$\text{et } (1 + e^{-2x})(e^{2x} - 1)$$

$$= e^{2x} - e^{-2x}$$

$$\text{also } (1 - e^{-2x})(e^{2x} + 1) = (1 + e^{-2x})(e^{2x} - 1)$$

$$\text{et } \forall x \in \mathbb{R}, \frac{1 - e^{-4x}}{1 + e^{2x}} = \frac{e^{2x} - 1}{e^{4x} + 1}$$

3) Soit $x \in \mathbb{R}$.

$$(e^x + e^{-x})^2 - 2 = \frac{e^{4x} + 1}{e^{2x}}$$

$$\begin{aligned} \Rightarrow (e^x)^2 + 2e^x e^{-x} + (e^{-x})^2 - 2 \\ = \frac{e^{4x} + 1}{e^{2x}} \end{aligned}$$

$$\Leftrightarrow e^{2x} + 2 + e^{-2x} - 2 = \frac{e^{4x} + 1}{e^{2x}}$$

$$\Leftrightarrow e^{2x} + e^{-2x} = \frac{e^{4x} + 1}{e^{2x}}$$

$$\Rightarrow e^{2x} (e^{2x} + e^{-2x}) = e^{4x} + 1$$

$$\Leftrightarrow e^{4x} + e^0 = e^{4x} + 1$$

$$\Leftrightarrow e^{4x} + 1 = e^{4x} + 1$$



exercice :

Soit $x, y \in \mathbb{R}$

$$x \geq y$$

$$g(x) \geq g(y)$$

$$f(g(x)) \geq f(g(y))$$

$$\text{donc } (f \circ g)(x) \geq (f \circ g)(y)$$

donc $f \circ g$ croissante.