

Exercices

exercice 2:

a) $u_n = \frac{1}{n+1}$

$$u_0 = 1$$

$$u_1 = \frac{1}{2}$$

$$u_2 = \frac{1}{3}$$

$$\bullet u_1 - u_0 = \frac{1}{2} - 1 = -\frac{1}{2}$$

$$\bullet u_2 - u_1 = \frac{1}{3} - \frac{1}{2} = -\frac{1}{6}$$

donc (u_n) n'est pas arithmétique.

$$b) v_n = (n+1)^2 - n^2$$

$$v_0 = 1^2 - 0^2 = 1$$

$$v_1 = 4 - 1 = 3$$

$$v_2 = 9 - 4 = 5$$

$$\cdot v_1 - v_0 = 3 - 1 = 2$$

$$\cdot v_2 - v_1 = 5 - 3 = 2$$

Montrons que (v_n) est arithmétique

$$v_{n+1} = (n+2)^2 - (n+1)^2$$

$$= n^2 + 4n + 4 - (n^2 + 2n + 1)$$

$$= \cancel{n^2} + 4n + 4 - \cancel{n^2} - 2n - 1$$

$$= 2n + 3 = (2n + 1) + 2$$

$$\alpha \quad v_n = (n+1)^2 - n^2$$

$$= \cancel{n^2} + 2n + 1 - \cancel{n^2}$$

$$= 2n + 1.$$

Donc $v_{n+1} = v_n + 2$

et (v_n) est arithmétique
de raison 2.

exercice 6 :

1) La suite (u_n) est arithmétique de raison de 8

2) Comme la raison $8 > 0$, alors la suite est croissante.

3)
$$u_n = u_0 + 8 \times n$$

On a $u_n = 118 + 8n$,
pour tout $n \in \mathbb{N}$.

on cherche $u_4 = 118 + 8 \times 4$

$$= 118 + 32$$
$$= 150$$

donc en 2024 il y aura
150 adhérents au club.

4) $u_n = 118 + 8n$, pour
tout $n \in \mathbb{N}$.

5) $118 + 8n > 200$

$$\Leftrightarrow 8n > 200 - 118$$

$$\Leftrightarrow 8n > 82$$

$$\Leftrightarrow n > \frac{82}{8} = 10,25$$

donc le plus petit entier
 n tel que $118 + 8n > 200$
est $n = 11$.

6) ① D'après la question
le club accueillera au
moins 200 adhérents en
2031.

propriété 3.1 :

$$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

preuve : $2 \times (1 + \dots + n)$

The diagram illustrates the proof by doubling the sum. It shows two rows of terms: the top row is $1 + 2 + \dots + (n-1) + n$ and the bottom row is $n + (n-1) + \dots + 2 + 1$. A blue bracket groups both rows together. Colored ovals highlight the pairing of terms: a red oval around the 1 in the top row and the n in the bottom row; a green oval around the 2 in the top row and the (n-1) in the bottom row; a purple oval around the (n-1) in the top row and the 2 in the bottom row; and a green oval around the n in the top row and the 1 in the bottom row. This visualizes that each pair of terms sums to $n+1$.

$$\begin{aligned} & (n+1) + (n+1) + \dots + (n+1) + (n+1) \\ &= n \times (n+1) \end{aligned}$$

Donc :

$$2 \times (1 + \dots + n) = n(n+1)$$

Ainsi : $1 + \dots + n = \frac{n(n+1)}{2}.$

exemple 3.1 :



propriété 3.1 :

$$u_0 + \dots + u_n = (n+1) \times \left(\frac{u_0 + u_n}{2} \right)$$

exemple 3.2 :

$$u_0 = 5 \quad \text{et} \quad u_{10} = 5 + 3 \times 10 \\ = 35$$

donc :

$$S = u_0 + \dots + u_{10} \\ = (10 + 1) \times \left(\frac{5 + 35}{2} \right)$$

$$= 11 \times \frac{40}{2} = 220$$

exercice 12:

$$\begin{aligned} 1) R &= 0 + 1 + 2 + \dots + 100 \\ &= 1 + 2 + \dots + 100 \\ &= \frac{100 \times 101}{2} \\ &= 50 \times 101 \\ &= 5050 \times 10 \\ &= 50500 \end{aligned}$$

$$\begin{aligned} 2) S &= 0 + 2 + 4 + \dots + 100 \\ &= 2 \times (0 + 1 + 2 + \dots + 50) \\ &= 2 \times (1 + 2 + \dots + 50) \end{aligned}$$

$$= \cancel{2} \times \frac{50 \times 51}{\cancel{2}}$$

$$= 50 \times 51$$

$$= 2550$$

$$3) T = 1 + 3 + 5 + \dots + 99$$

$$\text{or } R = S + T$$

$$\text{done } T = R - S$$

$$= 5050 - 2550$$

$$= 2500.$$