

Feuille 4

exercice 1:

$$f(x) = \underbrace{\cos(\sin(x))}_{u(v(x))}$$

$$u(x) = \cos(x)$$

$$u'(x) = -\sin(x)$$

$$v(x) = \sin(x)$$

$$v'(x) = \cos(x)$$

$$(u(v(x)))' = v'(x) \times u'(v(x))$$

$$\begin{aligned} f'(x) &= \cos(x) \cdot x - \sin(\sin(x)) \\ &= -\cos(x) \sin(\sin(x)) \end{aligned}$$

• $f(x) = \sin\left(\frac{1}{x}\right)$

$$u(x) = \sin(x)$$

$$u'(x) = \cos(x)$$

$$v(x) = \frac{1}{x}$$

$$v'(x) = -\frac{1}{x^2}$$

$$f'(x) = -\frac{1}{x^2} \cdot x \cos\left(\frac{1}{x}\right)$$

$$\cdot f(x) = \underbrace{\tan(\sin(h(x)))}_{u(v(x))}$$

$$u(x) = \tan(x)$$

$$u'(x) = 1 + \tan^2(x)$$

$$v(x) = \frac{\sin(h(x))}{g(h(x))}$$

$$\begin{aligned} \hookrightarrow g(x) &= \sin(x) / g'(x) = \cos(x) \\ h(x) &= h(x) / h'(x) = \frac{1}{x} \end{aligned}$$

$$v'(x) = \frac{1}{x} x \cos(h(x))$$

$$f'(x) = \frac{1}{x} \cos(h(x)) \left(1 + \tan^2(\sin(h(x))) \right)$$

$$\begin{aligned}
 f(x) &= \cos(\sqrt[3]{h(x)}) \\
 &= \underbrace{\cos(h(x)^{\frac{1}{3}})}_{u(v(x))}
 \end{aligned}$$

$$f' = \cdot^{\frac{1}{3}}$$

$$u(x) = \cos(x)$$

$$u'(x) = -\sin(x)$$

$$v(x) = (h(x))^{\frac{1}{3}}$$

$$(u^n)' = n u' u^{n-1}$$

$$v'(x) = \frac{1}{3} \times \frac{1}{x} \times (h(x))^{-\frac{2}{3}}$$

$$f'(x) = \frac{1}{3} \times \frac{1}{x} \times h(x)^{-\frac{2}{3}} - \sin(h(x)^{\frac{1}{3}})$$

$$y^x = e^{x \ln(y)}$$

$$\begin{aligned} \bullet \quad f(x) &= x^{\frac{1}{x}} , \quad x > 0 \\ &= \underbrace{e^{\frac{1}{x} \ln(x)}}_{e^u} \end{aligned}$$

$$(e^u)' = u' e^u$$

$$u(x) = \underbrace{\frac{1}{x} \ln(x)}_{u'}$$

$$u'(x) = -\frac{1}{x^2} \ln(x) + \frac{1}{x} \times \frac{1}{x}$$

$$= -\frac{1}{x^2} \ln(x) + \frac{1}{x^2}$$

$$= \frac{1}{x^2} (1 - \ln(x))$$

$$f'(x) = \frac{1}{x^2} (1 - \ln(x)) e^{\frac{1}{x} \ln(x)}$$

$$= \frac{1}{x^2} (1 - \ln(x)) x^{\frac{1}{x}}.$$

exercice 2 :

règle de l'Hospital :

• si $\lim_{x \rightarrow a} f(x) = 0 = \lim_{x \rightarrow a} g(x)$
 $\mathbb{R} \text{ ou } \pm \infty$ $\mathbb{R} \text{ ou } \pm \infty$

et si $\lim_{x \rightarrow a} \frac{f'(x)}{g'(x)} = \underline{p}$
 $\mathbb{R} \text{ ou } \pm \infty$

alors $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = p.$

• si $\lim_{x \rightarrow a} g(x) = \pm \infty$ et $\lim_{x \rightarrow a} f(x)$

$\lim_{x \rightarrow a} \frac{f'(x)}{g'(x)} = p$, alors $\lim_{x \rightarrow a} \frac{f}{g} = p$

$$\lim_{x \rightarrow 1} \frac{\sqrt[3]{x} - 1}{x^4 - 1}$$

$$\lim_{x \rightarrow 1} f(x) = 0$$

$$\lim_{x \rightarrow 1} g(x) = 0$$

$$\begin{aligned} \sqrt[3]{x} &= x^{\frac{1}{3}} \\ (x^{\frac{1}{3}})' &= \frac{1}{3}x^{\frac{1}{3}-1} \end{aligned}$$

$$\lim_{x \rightarrow 1} \frac{f'(x)}{g'(x)} = \lim_{x \rightarrow 1} \frac{\frac{1}{3}x^{-\frac{2}{3}}}{4x^3}$$

$$\frac{x^a}{x^b} = x^{a-b}$$

$$= \lim_{x \rightarrow 1} \frac{1}{12} \frac{x^{-\frac{2}{3}-3}}{x^{\frac{11}{3}}}$$

$$= \lim_{x \rightarrow 1} \frac{1}{12} \times \frac{1}{x^{\frac{11}{3}}}$$

$$= \frac{1}{12}$$

done $\lim_{x \rightarrow 1} \frac{\sqrt[3]{x} - 1}{x^3 - 1} = \frac{1}{12}$

$(ru)'$

$\lim_{x \rightarrow +\infty} \frac{\sqrt{1+x} - 2}{\sqrt{1+x^2}}$ Jf Jg

$\lim_{x \rightarrow +\infty} f(x) = +\infty$

$\lim_{x \rightarrow +\infty} g(x) = +\infty$

$$\frac{f'(x)}{g'(x)} = \frac{\frac{1}{2\sqrt{1+x}}}{\frac{x}{2\sqrt{1+x^2}}} = \frac{\frac{1}{2\sqrt{1+x}}}{\frac{x}{\sqrt{1+x^2}}}$$

$$= \frac{1}{2\sqrt{1+x}} \times \frac{\sqrt{1+x^2}}{x}$$

$$= \frac{\sqrt{1+x^2}}{2x\sqrt{1+x}}$$

$$= \frac{\sqrt{x^2 \left(\frac{1}{x^2} + 1 \right)}}{2x \sqrt{x \left(\frac{1}{x} + 1 \right)}}$$

$$\begin{aligned} & \frac{x^2 + 1}{x + 1} \\ &= \frac{x^2 \left(1 + \frac{1}{x^2} \right)}{x \left(1 + \frac{1}{x} \right)} \\ &= \frac{1}{x} \times \frac{1 + \frac{1}{x^2}}{1 + \frac{1}{x}} \end{aligned}$$

$$= \frac{\sqrt{x^2} \sqrt{\frac{1}{x^2} + 1}}{2x \sqrt{x} \sqrt{\frac{1}{x} + 1}}$$

$$= \frac{\cancel{x} \sqrt{\frac{1}{x^2} + 1}}{2\cancel{x} \sqrt{x} \sqrt{\frac{1}{x} + 1}}$$

→ 0

$$= \frac{1}{2\sqrt{x}} \times \frac{\sqrt{\frac{1}{x^2} + 1}}{\sqrt{\frac{1}{x} + 1}} \rightarrow 1$$

$\xrightarrow{x \rightarrow +\infty} 0$

$$\xrightarrow{x \rightarrow +\infty} 0$$