

Algèbre Linéaire

exercice 2:

$$A_3 = \begin{pmatrix} 1 & 0 & 2 \\ 2 & 4 & -1 \\ -2 & 0 & 2 \end{pmatrix}$$

$$A^{-1} = \frac{1}{\det A} {}^E(\text{com}(A))$$

$${}^E A_3 = \begin{pmatrix} 1 & 2 & -2 \\ 0 & 4 & 0 \\ 2 & -1 & 2 \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 2 & 4 & -1 & 0 & 1 & 0 \\ -2 & 0 & 2 & 0 & 0 & 1 \end{array} \right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 4 & -5 & -2 & 1 & 0 \\ 0 & 0 & 6 & 2 & 0 & 1 \end{array} \right)$$

$L_2 \leftarrow L_2 - 2L_1$
 $L_3 \leftarrow L_3 + 2L_1$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 4 & -5 & -2 & 1 & 0 \\ 0 & 0 & 1 & \frac{11}{6} & 0 & \frac{1}{6} \end{array} \right)$$

$L_3 \leftarrow \frac{1}{6}L_3$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{3} & 0 & -\frac{1}{6} \\ 0 & 4 & 0 & \frac{1}{3} & 1 & \frac{5}{6} \\ 0 & 0 & 1 & \frac{1}{3} & 0 & \frac{1}{6} \end{array} \right) \begin{array}{l} L_1 \\ \leftarrow L_1 - 2L_3 \\ L_2 \\ \leftarrow L_2 + 9L_3 \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{3} & 0 & -\frac{1}{6} \\ 0 & 1 & 0 & \frac{1}{12} & \frac{1}{4} & \frac{5}{24} \\ 0 & 0 & 1 & \frac{1}{3} & 0 & \frac{1}{6} \end{array} \right) L_2 \leftarrow \frac{1}{4} L_2$$

$$\Rightarrow A_3^{-1} = \frac{1}{3} \begin{pmatrix} 1 & 0 & -1 \\ \frac{1}{4} & 0 & \frac{1}{6} \\ \frac{1}{3} & 1 & \frac{1}{6} \end{pmatrix}$$

Exercise 5:

$$A_2 = \begin{pmatrix} \overset{a_{11}}{2} & \overset{a_{12}}{3} \\ \underset{a_{21}}{3} & \underset{a_{22}}{4} \end{pmatrix}$$

$$\begin{pmatrix} a_{1j} \\ i+j \\ i,j \end{pmatrix}$$

$$A_3 = \begin{pmatrix} \overset{a_{11}}{2} & \overset{a_{12}}{3} & \overset{a_{13}}{4} \\ \underset{a_{21}}{3} & \underset{a_{22}}{4} & \underset{a_{23}}{5} \\ \underset{a_{31}}{4} & \underset{a_{32}}{5} & \underset{a_{33}}{6} \end{pmatrix}$$

Espaces vectoriels.

Soit E un ev.

méthode: montrer que F est un sev de E

$$\rightarrow 0_E \in F$$

$$\rightarrow \forall x, y \in F, \quad x + y \in F$$

$$\rightarrow \forall \lambda \in \underbrace{\mathbb{K}}_{\mathbb{R} \text{ ou } \mathbb{C}}, \quad \forall x \in F, \quad \lambda x \in F$$

exercice 2:

$$E_1 = \{(\omega, \bar{\omega}, 0) : \omega \in \mathbb{C}\}$$

Montrons qu'on E_1 est

un rev de $\mathbb{C}^3$.

- $O_{\mathbb{C}^3} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \in E_1$?

on pose $w = 0$, alors $\bar{w} = 0$.

Donc $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} w \\ \bar{w} \\ 0 \end{pmatrix} \in E_1$ ✓

- Soit $(w, \bar{w}, 0) \in E_1$ et
 $(z, \bar{z}, 0) \in E_1$

$$(w, \bar{w}, 0) + (z, \bar{z}, 0)$$

$$= (w + z, \bar{w} + \bar{z}, 0 + 0)$$

$$= (w + z, \overline{w + z}, 0)$$

donc en posant $x = w + z$,
 $(w, \bar{w}, 0) + (z, \bar{z}, 0) = (x, \bar{x}, 0)$
 $\in E_1$ ✓

• Soit $\lambda \in \mathbb{C}$ et $(\omega, \bar{\omega}, 0) \in E_1$

$$\lambda(\omega, \bar{\omega}, 0)$$

$$= (\lambda\omega, \lambda\bar{\omega}, \lambda \times 0)$$

$$= (\lambda\omega, \overline{\lambda\omega}, 0)$$

car $\lambda \in \mathbb{C}, \bar{\lambda} \neq \lambda$



on prend $\lambda = i \in \mathbb{C}$ et $(2i, -2i, 0) \in E_1$

$$i(2i, -2i, 0)$$

$$= (-2, 2, 0) \notin E_1$$

(Note: In the original image, the first -2 is underlined with a green bracket labeled ω , and the second 2 is underlined with a green bracket labeled $\bar{\omega}$ with a red X over it.)

donc E_1 n'est pas un
sev de $\mathbb{C}^3$.

sg: E_1 est un sev de $\mathbb{R}^3$

$\lambda(\omega, \bar{\omega}, 0)$, avec $\lambda \in \mathbb{R}$

$$= (\lambda\omega, \lambda\bar{\omega}, \lambda \cdot 0)$$

$$= (\lambda\omega, \overline{\lambda\omega}, 0)$$

↪ vraie car $\overline{\lambda} = \lambda$