

Changement de variables

$$I = \int_1^4 \frac{1-\sqrt{t}}{\sqrt{t}} dt$$

on pose $x = \sqrt{t}$

$$t = 1 \Rightarrow x = \sqrt{1} = 1$$

$$t = 4 \Rightarrow x = \sqrt{4} = 2$$

$$x = \sqrt{t} \Rightarrow dx = \frac{1}{2\sqrt{t}} dt$$

$$\Rightarrow 2\sqrt{t} dx = dt$$

$$\Rightarrow 2x dx = dt$$

$$I = \int_1^2 \frac{1-x}{x} \times 2x \, dx$$

$$= \int_1^2 2(1-x) \, dx$$

$$= 2 \int_1^2 1-x \, dx$$

$$= 2 \left[x - \frac{x^2}{2} \right]_1^2$$

$$= 2 \left[\left(2 - \frac{4}{2} \right) - \left(1 - \frac{1}{2} \right) \right]$$

$$= 2 \left[0 - \frac{1}{2} \right] = -1.$$

$$\bullet I = \int_1^e \frac{dt}{t \ln(t) + t}$$

on pose $x = \ln(t)$

$$t = 1 \Rightarrow x = \ln(1) = 0$$

$$t = e \Rightarrow x = \ln(e) = 1$$

$$dx = \frac{dt}{t} \Leftrightarrow t dx = dt$$

$$\Leftrightarrow e^x dx = dt$$

$$x = \ln(t) \Leftrightarrow t = e^x$$

$$I = \int_0^1 \frac{e^x dx}{2e^x x + e^x}$$

$$= \int_0^1 \frac{\cancel{e^x} dx}{\cancel{e^x} (2x + 1)}$$

$$= \int_0^1 \frac{1}{2x + 1} dx \quad \left[2x \times \frac{1}{2} = \frac{1}{2} u' \right]$$

$$= \left[\frac{1}{2} \ln(2x + 1) \right]_0^1$$

$$= \frac{1}{2} \ln(3) - \frac{1}{2} \cancel{\ln(1)} = \frac{1}{2} \ln(3)$$

$$\bullet \quad I = \int_0^1 \frac{dt}{1+e^t}$$

on pose $x = e^t$

$$dx = e^t dt$$

$$\Rightarrow \frac{dx}{e^t} = dt$$

$$\Rightarrow \frac{dx}{x} = dt$$

$$I = \int_1^e \frac{1}{1+x} \frac{dx}{x}$$

$$= \int_1^e \frac{1}{x(x+1)} dx$$

$$= \int_1^e \frac{1}{x} - \frac{1 \text{Ju}'}{x+1 \text{Ju}} dx$$

$$= \left[\ln(x) - \ln(x+1) \right]_1^e$$

$$= (\ln(e) - \ln(e+1)) - (\ln(1) - \ln(2))$$

$$= \ln(e) - \ln(e+1) + \ln(2)$$

$$= 1 - \ln(e+1) + \ln(2)$$