

Exercise 1 =

1. $f(x) = 6x^5$

$$F(x) = 6 \frac{x^6}{6}$$

$$= 1 \times x^6 = x^6 \quad \checkmark$$

2. $f(x) = 4x^3 - 7x + 5$

$$F(x) = 4 \frac{x^4}{4} - 7 \frac{x^2}{2} + 5x$$

$$= x^4 - \frac{7}{2}x^2 + 5x \quad \checkmark$$

3. $f(x) = 3(2x^4 - x^2 + 1)$

$$= 6x^4 - 3x^2 + 3$$

$$F(x) = 6 \times \frac{x^5}{5} - 3 \frac{x^3}{3} + 3x$$

$$= \frac{6}{5}x^5 - x^3 + 3x \quad \checkmark$$

4. $f(x) = \frac{5}{x^2} + 2x$

$$= 5x^{-2} + 2x^{\frac{2}{2}}$$

$$F(x) = \frac{5x^{-1}}{-1} + x^2$$

$$= -5x^{-1} + x^2 = -\frac{5}{x} + x^2$$

$$\frac{1}{x^2} \rightarrow -\frac{1}{x}$$

5. $f(x) = \frac{4}{\sqrt{x}} - \frac{3}{x}$

$$= 4x^{-\frac{1}{2}} - 3x^{-1}$$

$$= 4 \cdot x^{\frac{-1}{2}} - 3 \times \frac{x}{x}$$

$$= 8x^{\frac{1}{2}} - 3$$

$$\frac{1}{\sqrt{x}} \rightarrow 2\sqrt{x}$$

$$\frac{1}{2} \times \frac{2}{1} = \frac{2}{2} = 1$$

$$6. f(x) = 5e^x - 3x^2$$

$$F(x) = 5e^x - 3 \frac{x^3}{3}$$

$$= 5e^x - x^3$$

$$7. f(x) = 2(e^x + \cos x)$$

$$= 2e^x + 2\cos(x)$$

$$F(x) = 2e^x + 2 \times \sin(x)$$

$$8. f(x) = 7\cos x - 4\sin x$$

$$F(x) = 7x\sin(x) - 4x - \cos(x)$$

$$= 7\sin x + 4\cos x$$

$$9. f(x) = 3x^2 + 5\sin(x) - 2e^x$$

$$F(x) = 3 \frac{x^3}{3} + 5x - \cos x - 2e^x$$

$$= x^3 - 5\cos x - 2e^x$$

$$10. f(x) = -4(x^3 - 2\cos x + \sin x)$$

$$= -4x^3 + 8\cos x - 4\sin x$$

$$F(x) = -4 \frac{x^4}{4} + 8x\sin x - 4x\cos$$

$$= -x^4 + 8\sin x + 4\cos$$

Exercice 2 =

$$u' u^n \rightarrow \frac{u^{n+1}}{n+1}$$

1. $f(x) = 2x(x^2+3)^4 \rightarrow u' = 2x$ donc $u'u^n$ ✓
 $f(u) = \frac{1}{5} x (x^2+3)^5$ ✓

2. $f(x) = (3x+1)^5 = \frac{1}{5} \times 5(3x+1)^4 \times 3$
 $\rightarrow u' = 3$ donc $u'u^n$
 $f(u) = \frac{1}{3} \times \frac{1}{6} \times (3x+1)^6$
 $\frac{1}{18} = \frac{1}{6} \times (3x+1)^6$

3. $f(x) = \frac{2x}{x^2+4} \rightarrow u' = 2x$ donc $\frac{u'}{u}$

$f(u) = \ln(x^2+4)$ ✓

4. $f(x) = \frac{1}{5x-2} \rightarrow u' = 5$ ✓

$g(u) = \frac{1}{5} \times \frac{5}{5x-2}$ $\frac{u'}{u} \rightarrow \ln(u)$

$f(x) = \frac{1}{5} \times \ln(5x-2)$ ✓

5. $f(x) = 2x e^{x^2} \rightarrow u' = 2x$ donc $u' \times e^u \rightarrow e^u$

$f(u) = e^{x^2}$ ✓

6. $f(x) = e^{4x+1} \rightarrow u' = 4$ ✓

$g(u) = \frac{1}{4} \times 4 e^{4x+1} \rightarrow u' \times e^u \rightarrow e^u$

$f(u) = \frac{1}{4} e^{4x+1}$

$$7. f(x) = \frac{2^{\frac{1}{2}}}{\sqrt{x^2+1}}$$

$$u' = 2x$$

$$u = x^2 + 1$$

$$= \frac{1}{2} \times \frac{2x}{\sqrt{x^2+1}} \quad \frac{u'}{\sqrt{u}} \rightarrow 2\sqrt{u}$$

$$F(x) = \frac{1}{2} \times 2\sqrt{x^2+1}$$

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17) $\int_0^1 \underbrace{(x-3)}_{\text{dérivée}} e^x \, dx$ ← primitive

• $x e^x$: x dérive e^x primitive

• $x D_n(x)$: x primitive D_n dérive

• $x \begin{cases} \cos x \\ \sin x \end{cases}$: x dérive $\cos/\sin$ primitive

$= \left[\underbrace{(x-3)x}_{\text{primitive}} e^x \right]_0^1 - \int_0^1 \underbrace{e^x}_{\text{dérive}} x \, dx$

$$\int_a^b \underbrace{u}_{\text{dérive}} \times \underbrace{v}_{\text{primitive}} dx$$

$$= \left[u \times v \right]_a^b - \int_a^b v \times u' dx$$

$$= \left((1-3)e^1 - (0-3)e^0 \right)$$

$$- \int_0^1 e^x dx$$

primitive

$$= -2e + 3 - \left[e^x \right]_0^1$$

$$= -2e + 3 - (e^1 - e^0)$$

$$\int_a^b f(x) dx = \left[F(x) \right]_a^b = (F(b) - F(a))$$

$$= -2e + 3 - e + 1$$

$$= -3e + 4$$

18) $\int_0^{\pi} \underbrace{x}_{\text{derivative}} \underbrace{\cos(x)}_{\text{primitive}} dx$

$$= \left[x \sin(x) \right]_0^{\pi} - \int_0^{\pi} \sin(x) x dx$$

$$= \left(\pi \sin \pi - 0 \times \sin 0 \right)$$

$$- \int_0^{\pi} \sin(x) \, dx$$

primitive

$$= \left(\pi \times 0 - 0 \times 0 \right) - \left[-\cos(x) \right]_0^{\pi}$$

$$= - \left(-\cos(\pi) - (-\cos(0)) \right)$$

$$= - \left(-(-1) - (-1) \right)$$

$$= - (1 + 1) = -2$$

x	$0/2\pi$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\pi/-\pi$
cos	1	$\sqrt{3}/2$	$\sqrt{2}/2$	1/2	0	-1
sin	0	1/2	$\sqrt{2}/2$	$\sqrt{3}/2$	1	0

