

Ensembles et applications

exercice 2:

1) $E = \{1, 2, 3, 4, 5, 6\}$

$$A = \{1, 3, 5\}$$

$$B = \{1, 2, 4, 5\}$$

$$\bullet A \cup B = \{1, 2, 3, 4, 5\}$$

$$\bullet A \cap B = \{1, 5\}$$

$$\bullet A \setminus B = \{3\}$$

$$\bullet B \setminus A = \{2, 4\}$$

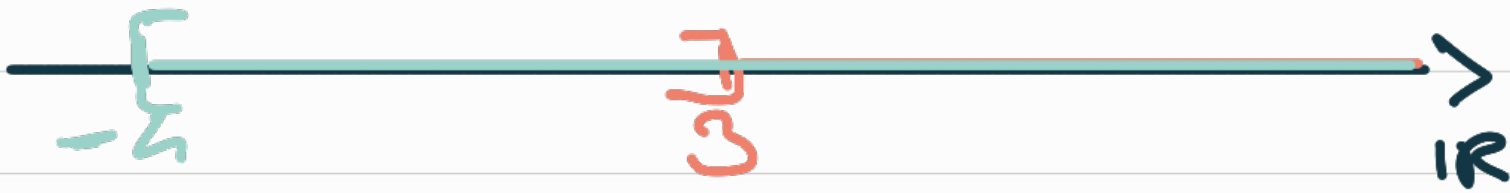
$$\bullet {}^c_E A = \{2, 4, 6\}$$

$$2) E = \mathbb{R} \quad A =]3; +\infty[$$

$$B = [-4; +\infty[$$

B

A



$$\bullet A \cup B = [-4; +\infty[$$

$$\bullet A \cap B =]3; +\infty[$$

$$\bullet A \setminus B = \emptyset$$

$$\bullet B \setminus A = [-4; 3]$$

$$\bullet {}^c A =]-\infty; 3]$$

exercice 3:

1) Montrer que :

$$A \cup B = B \iff A \subset B.$$

① $\implies$ On suppose avoir $A \cup B = B$. montrons que $A \subset B$

Soit $x \in A$

or $A \subset A \cup B$, donc

$$x \in A \cup B.$$

et par hypothèse $A \cup B = B$.

donc $x \in B$.

Donc $A \subset B$

$$A \subset A \cup B$$

$$B \subset A \cup B$$

$$A \cap B \subset A$$

$$A \cap B \subset B$$

$$x \in A \cup B$$

$$x \in A \text{ ou } x \in B$$

$$x \in A \cap B$$

$$x \in A \text{ et } x \in B$$

② $\Leftarrow$ On suppose $A \subset B$.

Montrons que $A \cup B = B$.

$\subseteq$ montrons que $A \cup B \subset B$

Soit $x \in A \cup B$

• soit $x \in A$, or par hypothèse

$A \subset B$, donc $x \in B$.

• Soit $x \in B$.

donc dans les deux $x \in B$.

Ainsi $A \cup B \subset B$.

2) De plus, on a toujours
 $B \subset A \cup B$.

Donc $A \cup B = B$

Conclusion :

$$A \cup B = B \iff A \subset B.$$

3) Montrer que :

$$A \setminus (A \cap B) = A \setminus B.$$

$$A \setminus B = A \cap \overline{A \cap B}$$

$$. A \setminus (A \cap B)$$

$$= A \cap G_e(A \cap B)$$

Pci de Morgan : $\overline{A \cap B} = \overline{A} \cup \overline{B}$

$$. \overline{A \cap B} = \overline{A} \cup \overline{B}$$

$$= A \cap (\overline{A} \cup \overline{B})$$

$$= (A \cap \overline{A}) \cup (A \cap \overline{B})$$

$$= \emptyset \cup (A \setminus B)$$

$$= A \setminus B$$

$$\emptyset \cup A = A$$

$$\emptyset \cap A = \emptyset$$

$$E \cup A = E$$

$$E \cap A = A$$

$$\begin{aligned} & \cdot A \cap (B \cup C) \\ &= (A \cap B) \cup (A \cap C) \end{aligned}$$

$$\begin{aligned} & \cdot A \cup (B \cap C) \\ &= (A \cup B) \cap (A \cup C) \end{aligned}$$

exercice 8:

$$A \Delta B = (A \cup B) \setminus (A \cap B)$$



$$2) . A \Delta A$$

$$= (A \cup A) \setminus (A \cap A)$$

$$= A \setminus A$$

$$= \emptyset$$

$$. A \Delta \emptyset$$

$$= (A \cup \emptyset) \setminus (A \cap \emptyset)$$

$$= A \setminus \emptyset$$

$$= A$$

$$\bullet A \Delta E$$

$$= (A \cup E) \setminus (A \cap E)$$

$$= E \setminus A$$

$$= C_E A$$

$$= \bar{A}$$

3) Montrer que :

$$A \Delta B = (A \cap \bar{B}) \cup (\bar{A} \cap B).$$

$$A \Delta B = (A \cup B) \setminus (A \cap B)$$

$$= (A \cup B) \cap (\overline{A \cap B})$$

$$\begin{aligned}
 &= (A \cup B) \cap (\bar{A} \cup \bar{B}) \\
 &= (A \cap \bar{A}) \cup (A \cap \bar{B}) \cup (B \cap \bar{A}) \\
 &\quad \cup (B \cap \bar{B})
 \end{aligned}$$

$$= \emptyset \cup (A \cap \bar{B}) \cup (B \cap \bar{A}) \cup \emptyset$$

$$= (A \cap \bar{B}) \cup (\bar{A} \cap B)$$

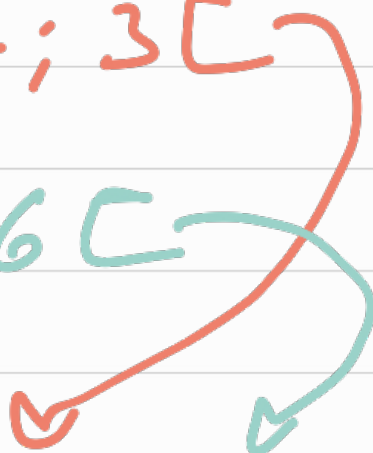
exercice 7 :

$$A \times B = \{ (x, y) \in \mathbb{R}^2 : x \in A, y \in B \}$$

1) $E = \{ (x, y) \in \mathbb{R}^2 : -1 < x < 3$
et $5 \leq y < 6 \}$.

$$x \in]-1; 3[$$

$$y \in [5; 6[$$



donc $E = A \times B$

avec $A =]-1; 3[$

$$B = [5; 6[.$$

$$2) E = \{(x, y) \in \mathbb{R}^2 : \\ 2 \leq x < 4 \text{ et } y = 3\}.$$

$$x \in [2; 4[$$

$$y \in \{3\}$$

$\{x\}$: singleton x

$$\text{donc : } E = A \times B$$

$$\text{avec } A = [2; 4[$$

$$B = \{3\}.$$